Integrated Modeling and Control of Electrostatic MEMS, Part II: Control

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Abstract—We present an integrated approach to modeling and control of electrostatically-actuated MEMS. The model consists of a set of fixed or movable electrodes, each coupled to the others through a resistor and capacitance, and to ground through a capacitance and a voltage or current source. This formulation incorporates movable structures, control electrodes, sense electrodes, and parasitic capacitances. The resulting dynamic equations may be directly incorporated into a control framework for stabilization and tracking. Formulas for two controllers are presented, one which requires measurement of electrode voltage and charge, the other which also requires measurement or estimation of the movable electrode velocities. In the absence of parasitics, the first controller can provide almost-global stabilization of any equilibrium configuration, while the second can do so as well, with additional improvement to the transient response. When parasitics are present, the approach must be modified to prevent the “charge pull-in” bifurcation. A general controller formulation for this case is still under development, but the paper presents a design for the 1 degree-of-freedom case.

I. INTRODUCTION

Electrostatic actuation of microelectromechanical systems (MEMS) makes use of the attractive coulomb forces that develop between capacitively-coupled conductors differing in voltage. Electrostatically-actuated MEMS are popular because they are simple in structure, flexible in operation, and may be fabricated from standard, well-understood, materials [8]. However, electrostatic actuation is highly nonlinear, making open-loop control over a large operating range difficult. Furthermore, the nonlinearity gives rise to a saddle-node bifurcation called “snap-through” or “pull-in” that results in operational limitations. Eliminating this effect would allow for enhanced functionality in a number of applications by increasing the operational range of the movable electrode, reducing the need for motion limiters and anti-stiction measures, and preventing disturbances from causing the movable electrode to depart from its stable operating region.

In [10] we have presented a series of results on global and semi-global stabilization of any point of the gap of an electrostatically actuated 1-DOF devices. These control algorithms can be implemented with static output feedback of the charge on the device and the voltage on the control electrode. Charge can be inferred from capacitance and voltage measurements, where the capacitance measurements are made at a frequency well above the natural frequencies of the device. It can also be obtained by placing an auxiliary electrode in series with the device, and measuring the voltage [2]. These ideas are extended in [9] by the authors to include more complex devices such as membranes and 6-DOF rigid structures and generalized in [13] to a broad class of electrostatically-actuated devices.

The model presented in [13] allows inter-electrode capacitive coupling but neglects inter-electrode resistive coupling. In part I of this paper we relax this assumption and allow for the possibility of leakage current flowing from one electrode to another. This extension to [13] allows more general modeling of parasitic effects. We stress that the results provide a general framework for modeling disturbance and control forces—including interelectrode coupling, fringing effects and parasitics—but that the effort needed to obtain explicit expressions in coordinates for a particular geometry may be significant.

In Section II, using the extended model but neglecting parasitics, we present two general control laws. One is based on energy-shaping ideas, and results in a static charge feedback controller, while the other is based on feedback-passivation techniques, and includes a velocity feedback term [5], [16]. The feedback-passivation based controller may be used to inject damping into the mechanical subsystem. Thus, if the transients of the system are under-damped, the second design should be used. However it is often the case that the generalized velocity cannot be measured directly. In [10], [11] we have proposed an observer to provide estimates of the generalized velocity given configuration measurements. Composing this result with the second design yields a dynamic output feedback controller that eliminates the pull-in bifurcation, stabilizes any equilibrium position, and improves transient performance. Depending on the properties of the zero dynamics of the system, the controllers may be locally, globally, or almost globally asymptotically stabilizing.

The controllers obtained in this paper in Section II include inter-electrode resistive coupling, but still neglect parasitic effects. In Section II-A we proceed to show how the modeling framework may be used to investigate the effects of parasitics on the stability properties of the controller. Unfortunately the
global properties of the two controllers of Section II cannot be guaranteed in the presence of parasitics. Addressing these effects in a general controller formulation is still an open question. However in section III we investigate a 1-DOF device with parasitic effects and show how the controllers may be modified so that every possible gap is stabilized. Finally we demonstrate the modified controllers using finite element ANSYS simulations that include fringing effects.

II. STABILIZATION OF ELECTROSTATICALLY ACTUATED MECHANICAL SYSTEMS

For the sake of completeness we first present the general model of an electrostatically actuated mechanical system derived in part I of this paper. The configuration of the mechanical system is denoted by \( g \), which is an element of the configuration space \( G \). Multiple moving electrodes are included by allowing \( G \) to be a product of configuration spaces \( G_\alpha \) where each \( G_\alpha \) corresponds to the configuration space of the moving electrode \( U_\alpha \). In this paper we assume that \( G_\alpha \) is finite dimensional. Specifically we assume that each of the configuration spaces \( G_\alpha \) of the movable electrodes is a finite dimensional Lie group. The mechanical system is assumed to satisfy the Euler-Lagrange equations with generalized forcing (the Lagrange-d’Alembert equations), where the forces are due to conservative mechanical terms, Rayleigh damping, and the electrostatic coupling derived in paper I. The potential energy of the mechanical subsystem (movable electrode) is \( U_g(g) \). Let \( f(g, \zeta) \) be the resultant conservative plus Rayleigh-type dissipative forces acting on the movable electrode. We remark that in this formulation inter-electrode mechanical coupling is considered by allowing the potential force to be derivable from a potential function defined on the entire configuration space \( G = \bigcup G_\alpha \). Then the combined model is given by

\[
\dot{Q} = \Theta C(g)^{-1} Q + \Lambda^{-1} u, \\
\dot{g} = g \cdot \zeta, \\
\dot{\zeta} = F^{-1}(ad^\zeta I g + f(g, \zeta) + f^e(g, Q)),
\]

(1) – (3)

where \( (g, \zeta) \in TG \simeq G \times \mathcal{G} \), \( f^e(g, Q) = \sum_{k=1}^n \frac{1}{2} \sigma \zeta \Lambda f_k(g)(Q + Q) - \Lambda K(Q - Q) \), \( f_k(g) = \sum_{n=m+p+1}^{N} f_k^n \) where we have assumed that the last \( q \) electrodes are free to move.

The \( f_k^n(g) \) are now the generalized electrostatic forces. The index \( n \) is the dimension of \( G \). The term \( ad^\zeta I \) describes the forces that arise from the non-Euclidean nature of \( G \). These are variously known as inertial or apparent forces and include, for example, coriolis terms. For further details on deriving this model we refer to part I of this paper.

For simplicity we assume that the movable \( q \) electrodes are grounded. Thus in computing the electrostatic field we assume that they too are a part of the grounded boundary \( \bar{S} \) of the system. Consider the mechanical part of the system given by (2) – (3). Assume that for a given desired \( (\bar{g}, 0) \) there exists a constant force \( f^s \) such that with \( f^s(g, Q) \equiv f^c \) the point \( (\bar{g}, 0) \) is at least a locally asymptotically stable equilibrium of the mechanical system and that there exists a \( Q \) such that \( f^c(\bar{g}, Q) = f^c \) holds. In effect we are assuming that by driving the entire charge \( Q \) to a feasible charge \( \bar{Q} \) the pull-in bifurcation can be eliminated. If full information of \( Q \) and \( V \) is available and if it is possible to control the charge on each of the electrodes then using the results of [13] it can be shown that both the charge feedback control

\[ u = -\Lambda^{-1} (\Theta V + K(Q - \bar{Q})) , \]

(4)

and the passivity based feedback

\[ u = -\Lambda \Theta V - \frac{1}{2} \sum_{k=1}^n \zeta \Lambda f_k(g)(Q + Q) - \Lambda K(Q - Q) , \]

(5)

where \( K > 0 \) is a positive definite matrix, at least locally asymptotically stabilizes \((Q, \bar{g}, 0)\). In both cases the convergence is almost global if the equilibrium \((\bar{g}, 0)\) of (2) – (3) is almost globally stable with \( f^s = f^c(g, 0) \). The implementation of the passive based control (5) requires the device velocity which can be estimated using a Riemannian observer as in [11]. Using the separation principle proved in [12], it can be shown that the convergence properties are preserved even if the velocity terms in (5) are replaced by their estimates. Both controls (4) and (5) are more general than that provided in [13] in the sense that they allow inter-electrode resistive coupling. However they still neglect parasitic effects.

A. Effects of Parasitics

Typically with parasitics, it is not possible to directly control or measure the charges on the parasitics nor is it possible to directly measure the voltages on the parasitic plates. Without loss of generality let the first \( m \) electrodes be the drive electrodes and the next \( p \) number of electrodes be the parasitic electrodes. Accordingly we partition \( Q, V, u, \Lambda, \Theta, C^{-1}(g) \) and \( f_k(g) \) as follows: \( Q = [Q^c \ Q^p]^T, V = [V_c^T \ V_p]^T, u = [u_c^T 0]^T \).

Using this notations the electrical sub-system (1) can be rewritten as

\[
\dot{Q}_c = \Theta_c V_c + \Theta_{cp} C_{cp} Q_c + \Theta_{cc} Q_c + \Lambda_{cc}^{-1} u_c , \]
\[
\dot{Q}_p = (\Theta_{pc} C_{cc} + \Theta_{pp} C_{cp}) Q_c + (\Theta_{pc} C_{cc} + \Theta_{pp} C_{cp}) Q_p .
\]

(6) – (7)

The voltage on the drive electrode, \( V_c = C_{cc} Q_c + C_{cp} Q_p \), and the charge, \( Q_c \), are assumed to be measured. The electrostatic force can also be re-written as \( f^e(g, Q) = \frac{1}{2} \sum_{k=1}^n (Q_k^c f_k^c Q_c + 2Q_k^p f_k^c Q_p + Q_k^p f_k^pp Q_p) \). The last two
terms are the parasitic effects on the electrostatic forces of the system 
\[ f^p(g, Q_c, Q_p) = \frac{1}{2} \sum_{k=1}^n (2Q^T f^p_\ell Q_p + Q^T f^p_\ell Q_p) \]
while 
\[ f^c (g, Q_c) = \frac{1}{2} \sum_{k=1}^n Q^T f^c_\ell Q_c \]
are the control electrode effects.

If electrical cross-coupling is also ignored then \( \Theta \) and \( \Lambda \) are diagonal and \( \Theta = -\Lambda^{-1} \). Then (6)–(7) and (2)–(3) reduce to
\[
\dot{Q}_c = -\Lambda^{-1} \left( C^{cc} (g) Q_c + C^{cpp} (g) Q_p - u_c \right) \quad (8) \\
\dot{Q}_p = -\Lambda^{-1} \left( C^{cpp} (g) Q_c + C^{cpp} (g) Q_p \right), \quad (9)
\]
\[
\dot{g} = g \cdot \zeta, \quad (10)
\]
\[
\zeta = I^{-1} \left( ad^2_x \zeta_f + f(g, \zeta) + f^c(g, Q_c) + f^p(g, Q_c, Q_p) \right) \quad (11)
\]
The controls (4) and (5) are derived neglecting the parasitic charge dynamics (9) and the parasitic forces \( f^p(g, Q_c, Q_p) \).

Let us investigate the implications of these neglected effects on the controls. From (9) it can be seen that at equilibrium 
\[ \ddot{Q}_c = \frac{1}{r_{cc}} \left( C^{cc} (g) Q_c + C^{cpp} (g) Q_p - u_c \right) \]
\[ \ddot{Q}_p = \frac{1}{r_{pp}} \left( C^{cpp} (g) Q_c + C^{cpp} (g) Q_p \right), \]
\[ \ddot{g} = 0 \]
\[ \zeta = I^{-1} \left( ad^2_x \zeta_f + f(g, \zeta) + f^c(g, Q_c) + f^p(g, Q_c, Q_p) \right) \quad (11)
\]
The effect of this on the equilibrium conditions could result in pull-in with respect to the charge \( Q_c \). Specifically the equilibrium configurations given by 
\[ f(\dot{g}, 0) + f^c(g, \dot{Q}_c) + f^c(g, \dot{Q}_c) + (C^{cpp})^{-1} C^{cpp} \dot{Q}_c = 0 \]
may bifurcate with respect to \( Q_c \), even if the equilibrium configurations of 
\[ f(\dot{g}, 0) + f^c(g, \dot{Q}_c) = 0 \]
may not. This phenomena is observed in [6] and is termed charge pull-in. If so both (4) and (5) will not be able to stabilize equilibria beyond pull-in. We demonstrate this with respect to a 1-DOF device in the next section. The solution to the problem lies in picking the correct output with respect to which the bifurcation can be eliminated. A generalization of this idea is not complete. However we demonstrate it on the 1-DOF device considered below and construct two controllers that will stabilize any feasible equilibrium configuration even in the presence of parasitics.

### III. Example: 1-DOF Piston Microactuators with Parasitics

Consider a device shown in figure 1. The two bottom electrodes \( S_1, S_2 \) are fixed while the top electrode \( S_3 \) is free to move. The zero voltage gap between the moving and drive voltage is \( d \) and the distance between the parasitic and drive electrode is \( \delta \). The moving electrode is considered to be grounded. Let \( \{e_1\} \) be a co-ordinate frame fixed at the center of mass of the moving electrode, when it is at the zero voltage equilibrium, with \( e_1 \) perpendicular to the moving electrode and pointing away from the fixed electrode. We assume that the moving electrode moves only in the \( e_1 \) direction. Thus the configuration space of the mechanical part of the system is \( G = R \). Denote the displacement of the moving electrode by the co-ordinate \( x \).

Let \( \frac{A_p}{A} = A_r \leq 1 \). From section IIIA of part I of this paper we have that the general corresponding governing equations of this model is given by
\[
\dot{Q}_c = -\frac{1}{r_{cc}} \left( x + d \right) \left( Q_c + Q_p \right) + \frac{1}{r_{cc}} u_c, \quad (12)
\]
\[
\dot{Q}_p = -\frac{1}{r_{pp}} \left( x + d \right) \left( Q_c + \left( A_r (x + d) + \delta \right) Q_p \right), \quad (13)
\]
\[
\dot{x} = v, \quad (14)
\]
\[
\dot{v} = -2\zeta \omega v - \omega^2 x - \frac{(Q_c + Q_p)^2}{2meA}, \quad (15)
\]
where for convenience we have set \( u_p \equiv 0 \). The voltage \( V_c = \frac{(x+d)}{\epsilon A} (Q_c + Q_p) \) on the drive electrode is assumed to be measured.

1) **Voltage and Charge Pull-In:** When \( u_c \equiv \ddot{u} \) the equilibrium gap is given by \( \frac{\epsilon A_2 v^2}{2m \omega^2} = -\ddot{x} (x + d)^2 \) and is the same as the case without the parasitics and hence the voltage pull-in bifurcation exists. Without the parasitics with charge control, i.e \( Q_c \equiv Q_e \), the equilibrium gap is given by \( \frac{\epsilon A_2 v^2}{2m \omega^2} = -\ddot{x} \) and clearly pull-in does not exist [10]. However with the parasitics the equilibrium equation for charge control becomes \( \frac{\epsilon A_2 v^2}{2m \omega^2} = -\ddot{x} (A_r (x + d) + \delta)^2 \) and it can be seen that the pull-in bifurcation persists. In fact it occurs when the gap is \( \frac{2A_r d - \delta}{3A_r} \) and is less than the voltage pull-in gap of \( \frac{2d}{\epsilon A} \). Furthermore it can also be seen that if \( \delta \geq 2A_r d \) then charge pull-in does not occur. Figure 2 shows the equilibrium curves for different values of the parameter \( \gamma = \frac{2A_r d}{A} \). For \( \gamma = 5 \) it can be seen that there corresponds two equilibria in the gap for a given \( Q_c \). It can be shown that the one corresponding to a larger gap is stable while the one corresponding to a smaller gap is unstable. Thus the two controls (4) and (5) will not almost globally stabilize every point in the gap. In what follows we show how one can overcome this by defining a new output with respect to which the bifurcation does not exists.
Let \( Q_e = Q_x + Q_p \). Then from equation (15) the equilibrium condition is given by \( \frac{Q_e^2}{2 \kappa A m \omega^2} = -\ddot{x} \) and the pull-in does not exist with \( Q_e \) control. Furthermore the system has relative degree one. Motivated by this we transform the system (12) - (15) from \( (Q_e, Q_p, x, v) \) to \( (Q_e, Q_p, x, v) \) and apply the input output linearizing control

\[
u = \frac{1}{2 \kappa A} (Q_e + \bar{Q}_e) v - k (Q_e - \bar{Q}_e)
\]

and obtain the resulting input output linearized system given by

\[
\dot{Q}_e = \nu,
\]

\[
\dot{x} = v,
\]

\[
\ddot{x} = -2 \zeta \omega \dot{x} - \omega^2 x - \frac{1}{2 m \kappa A} Q_e^2,
\]

\[
\dot{Q}_p = -\frac{\delta}{r_{pp} \kappa A} Q_p - \frac{(x + d)}{r_{pp} \kappa A} Q_e.
\]

In the expression of the control (16) we have used the fact that \( V_c = \frac{x + d}{r_{pp} \kappa A} (Q_p + Q_e) = \frac{x + d}{r_{pp} \kappa A} Q_e \). \( Q_e = \frac{\epsilon A}{x + d} V_c \) and \( Q_p = Q_e - Q_c = \frac{\epsilon A}{x + d} V_c - Q_c \).

Observe that equations (17) - (19) are in a form that corresponds exactly to that treated by the authors in [10], [13]. Thus enabling those results to be applied to these equations without modification. Specifically, under suitable contact assumptions, this provides two globally asymptotically stabilizing controls schemes. One of the strategies is based on charge based feedback that results in

\[
\nu = -k (Q_e - \bar{Q}_e) = -k \left( \frac{\epsilon A}{x + d} V_c - \bar{Q}_e \right),
\]

where \( k > 0 \), while the other employs velocity feedback and is shown to improve transient performance for under-damped systems. This second control is given by

Fig. 3. The vertical displacement for the charge feedback control (21) in the presence of parasitics.

Fig. 4. The vertical displacement for the velocity feedback control (22) in the presence of parasitics.

IV. Conclusion

We present a general modeling framework for the analysis of electrostatically actuated electromechanical devices. Within this framework we derive two control laws, one a static output feedback for almost-global stabilization, the other a dynamic output feedback controller which additionally improves transient performance. We also use the model to explore the effects of parasitics. In general the stability properties of the controllers can not be guaranteed. But in the 1-DOF case we show how they can be recovered by the appropriate modifications.

REFERENCES


