Two-Time Scale Force and Position Control of Flexible Manipulators

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Abstract
The problem of controlling force and position of a flexible link manipulator in contact with a compliant environment is considered in this paper. Using singular perturbation theory, a slow subsystem associated with rigid motion dynamics and a fast subsystem associated with link flexible dynamics are identified. Hence a two-time scale control strategy is adopted, consisting in a force and position control action for the slow subsystem and a stabilizing action for the fast subsystem. Both regulation and tracking problems are considered for the position, while the contact force is kept to a constant desired value. Simulations are presented for a two-link planar manipulator under gravity.

1 Introduction
Lightweight flexible manipulators offer a number of advantages with respect to conventional industrial robots in terms of high speed, large workspace and high payload-to-arm weight. The trade-off consists in a much more complex dynamics, due to the distributed flexibility of the links, which makes more difficult (and challenging) the control problem [1].

In case of interaction with the environment, it is required to control both the motion of the manipulator tip and the contact force. While several control schemes exist to force and position control of rigid robot manipulators [2], only few papers consider flexible manipulators. Stability problems are discussed in [3], while a hybrid position and force control approach is considered in [4, 5].

When the link stiffness is large, a two-time scale model of the flexible manipulator can be derived [6], consisting in a slow subsystem describing the rigid body motion and a fast subsystem describing the flexible motion. Hence a composite strategy can be adopted, based on a slow control designed for the equivalent rigid manipulator and a fast control which stabilizes the fast subsystem. Stability can be proven via singular perturbation theory. Perturbation techniques were used both in motion control problems ([6, 7, 8] and in force and position control problems ([4, 5, 9]).

In this paper a two-time scale force and position control for flexible manipulators is proposed, based on the parallel approach developed in [10, 11] for rigid robots in contact with compliant environments. As opposed to hybrid control strategies where force and position are controlled separately in reciprocal subspaces, both force and position are controlled in the full space. This makes parallel controllers suitable to manage contacts with non perfectly known environments and unplanned collisions.

Two different parallel control schemes are designed for the slow dynamics: the first ensures force and position regulation, while the second guarantees force regulation and position tracking. An additional control action is required in both cases to stabilize the fast dynamics related to link flexibility. The control laws are tested in simulation on the model of a two-link planar manipulator in contact with an elastically compliant plane.

2 Modelling
Consider a serial chain manipulator with $n$ flexible links connected by rigid revolute joints subject only to bending deformations in the plane of motion, without torsional effects. A sketch of a two link manipulator is shown in Fig. 1 with coordinate frame assignment. The rigid motion is described by the joint angles $\theta_i$, while $w_i(x_i)$ denotes the transversal deflection of link $i$ at $x_i$ with $0 \leq x_i \leq l_i$, being $l_i$ the link length.
A finite-dimensional model of link flexibility can be obtained by the assumed mode technique. The direct kinematics equation expressing the $(2 \times 1)$ position vector $p$ of the manipulator tip point as a function of the $(n \times 1)$ vector of the joint variables $\theta$ and the $(m \times 1)$ vector of the deflection variables $\delta$ can be written in the form [12]

$$p = k(\theta, \delta), \quad (1)$$

where $m = \sum_{i=1}^{n} m_i$, being $m_i$ the number of modes considered to express the deflection of link $i$. In view of (1), the differential kinematics equation expressing the tip velocity $\dot{p}$ as a function of $\dot{\theta}$ and $\dot{\delta}$ is

$$\dot{p} = J_\theta(\theta, \delta) \dot{\theta} + J_\delta(\theta, \delta) \dot{\delta}, \quad (2)$$

where $J_\theta = \partial k / \partial \theta$ and $J_\delta = \partial k / \partial \delta$.

Assume that the manipulator is in contact with the environment. By virtue of the virtual work principle, the vector $f$ of the forces exerted by the manipulator on the environment performing work on $p$ has to be related to the $(n \times 1)$ vector $J_\theta^T f$ of joint torques performing work on $\theta$ and the $(m \times 1)$ vector $J_\delta^T f$ of the elastic reaction forces performing work on $\delta$.

A finite-dimensional Lagrangian dynamic model of the planar manipulator in contact with the environment can be obtained in terms of the $n + m$ generalized coordinates $\theta, \delta$ in the form [12]:

$$\begin{bmatrix} B_{\theta\theta}(\theta, \delta) & B_{\theta\delta}(\theta, \delta) \\ B_{\delta\theta}^T(\theta, \delta) & B_{\delta\delta}(\theta, \delta) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} c_{\theta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \\ c_{\delta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} - \begin{bmatrix} J_\theta^T(\theta, \delta) f \\ J_\delta^T(\theta, \delta) f \end{bmatrix}, \quad (3)$$

where $B_{\theta\theta}, B_{\theta\delta}, B_{\delta\theta}$ are the blocks of the inertia matrix $B$, which is symmetric and positive definite, $c_{\theta}, c_{\delta}$ are the components of the vector of Coriolis and centrifugal forces, $g_{\theta}, g_{\delta}$ are the components of the vector of gravitational torques, $K$ is the diagonal and positive definite link stiffness matrix, $D$ is the diagonal and positive semidefinite link damping matrix, and $\tau$ is the vector of the input joint torques.

### 3 Two-time scale control

When the link stiffness is large, it is reasonable to expect that the dynamics related to link flexibility is much faster than the dynamics associated with the rigid motion of the manipulator so that the system naturally exhibits a two-time scale dynamic behaviour in terms of rigid and flexible variables. This feature can be conveniently exploited for control design.

Following the approach proposed in [6], the system can be decomposed in a slow and a fast subsystems by using singular perturbation theory; this leads to a composite control strategy for the full system based on separate control designs for the two reduced-order subsystems.

Assuming that full-state measurement is available and that a force sensor is mounted at the manipulator tip, the vector of joint torques can be conveniently chosen as

$$\tau = g_{\theta}(\theta, \delta) + J_\theta^T(\theta, \delta) f + u, \quad (4)$$

in order to cancel out the effects of the static torques acting on the rigid part of the manipulator dynamics; the vector $u$ is the new control input to be designed.

The time scale separation between the slow and fast dynamics can be determined by defining the singular perturbation parameter $\epsilon = 1 / \sqrt{k_m}$, where $k_m$ is the smallest coefficient of the diagonal stiffness matrix $K$, and the new variable

$$z = K \delta = \frac{1}{\epsilon^2} \hat{K} \delta, \quad (5)$$

corresponds to the elastic force, where $K = k_m \hat{K}$. Considering the inverse $H$ of the inertia matrix $B$, the dynamic model (3), with control law (4), can be rewritten in terms of the new variable $z$ as

$$\ddot{\theta} = H_{\theta\theta}(\theta, \epsilon^2 z) \left( u - c_{\theta}(\theta, \epsilon^2 z, \dot{\theta}, \epsilon^2 \dot{z}) \right)$$

$$- H_{\theta\delta}(\theta, \epsilon^2 z) \left( c_{\delta}(\theta, \epsilon^2 z, \dot{\theta}, \epsilon^2 \dot{z}) + g_{\delta}(\theta, \delta) \right)$$

$$+ \epsilon^2 D \hat{K}^{-1} \dot{z} + z + J_\theta^T(\theta, \delta) f$$

$$\epsilon^2 \ddot{z} = \hat{K} H_{\delta\delta}(\delta, \epsilon^2 z) \left( u - c_{\delta}(\theta, \epsilon^2 z, \dot{\theta}, \epsilon^2 \dot{z}) \right)$$

$$- \hat{K} H_{\delta\theta}(\delta, \epsilon^2 z) \left( c_{\delta}(\theta, \epsilon^2 z, \dot{\theta}, \epsilon^2 \dot{z}) + g_{\delta}(\theta, \delta) \right)$$

$$+ \epsilon^2 \hat{K}^{-1} \dot{z} + z + J_\delta^T(\theta, \delta) f, \quad (6)$$

where $H_{\theta\theta}, H_{\theta\delta}, H_{\delta\theta}, H_{\delta\delta}$ are the blocks of the inertia matrix $H$. The assumptions on the time scale separation allow the use of a fast and a slow control design.
where a suitable partition of $H$ has been considered

$$H = B^{-1} = \begin{bmatrix} H_{\theta \delta} & H_{\theta \delta s} \\ H_{\delta \delta}^T & H_{\delta \delta s} \end{bmatrix}. \quad (8)$$

Eqs. (6) and (7) represent a singularly perturbed form of the flexible manipulator model; when $\epsilon \to 0$, the model of an equivalent rigid manipulator is recovered. In fact, setting $\epsilon = 0$ and solving for $z$ in (7) gives

$$z_s = \tilde{H}_{\delta \delta s}^{-1}(\theta_s) \tilde{H}_{\delta \delta s}^T(\theta_s) \left( u_s - \tilde{c}_\theta(\theta_s, \dot{\theta}_s) \right) - \tilde{c}_\delta(\theta_s, \dot{\theta}_s) - g_\delta(\theta_s) - \tilde{J}_{s}^T(\theta_s) f_s, \quad (9)$$

where the subscript $s$ indicates that the system is considered in the slow time scale and the overbar denotes that a quantity is computed with $\epsilon = 0$. Plugging (9) into (6) with $\epsilon = 0$ yields

$$\dot{\theta}_s = \tilde{B}_{\theta \delta,\theta}(\theta_s) \left( u_s - \tilde{c}_\theta(\theta_s, \dot{\theta}_s) \right), \quad (10)$$

where the equality

$$\tilde{B}_{\delta \delta,\theta}(\theta_s) = \left( \tilde{H}_{\theta \delta}(\theta_s) - \tilde{H}_{\delta \delta}(\theta_s) \tilde{H}_{\delta \delta s}^{-1}(\theta_s) \tilde{H}_{\delta \delta s}^T(\theta_s) \right)$$

has been exploited, being $\tilde{B}_{\theta \delta,\theta}(\theta_s)$ the inertia matrix of the equivalent rigid manipulator and $\tilde{c}_\theta(\theta_s, \dot{\theta}_s)$ the vector of Coriolis and centrifugal torques.

The dynamics of the system in the fast time scale can be obtained by setting $\tau = t/\epsilon$, treating the slow variables as constants in the fast time scale, and introducing the fast variables $z_f = z - z_s$; thus, the fast system of (7) is

$$\frac{d}{d\tau} z_f = -K \tilde{H}_{\delta \delta s}(\theta_s) z_f + \tilde{K} \tilde{H}_{\delta \delta s}^T(\theta_s) u_f, \quad (12)$$

where the fast control $u_f = u - u_s$ has been introduced accordingly.

On the basis of the above two-time scale model, the design of a feedback controller for the system (6) and (7) can be performed according to a composite control strategy, i.e.,

$$u = u_s(\theta_s, \dot{\theta}_s) + u_f(z_f, dz_f/d\tau) \quad (13)$$

with the constraint that $u_f(0, 0) = 0$, so that $u_f$ is inactive along the equilibrium manifold specified by (9).

In order to design the slow control for the rigid nonlinear system (10), it is useful to derive the slow dynamics corresponding to the tip position. Differentiating Eq. (2) gives the tip acceleration

$$\ddot{p} = J_\theta(\theta, \delta) \ddot{\theta} + J_\delta(\theta, \delta) \ddot{\delta} + h(\theta, \delta, \dot{\theta}, \dot{\delta}), \quad (14)$$

where $h = J_\theta \theta + J_\delta \delta$; hence the corresponding slow system is

$$\ddot{p}_s = \tilde{J}_\theta(\theta_s) \bar{B}_{\theta \delta,\theta}(\theta_s) \left( u_s - \tilde{c}_\theta(\theta_s, \dot{\theta}_s) \right) + \tilde{h}(\theta_s, \dot{\theta}_s), \quad (15)$$

where Eq. (10) has been used. The slow dynamic models (10) and (15) enjoy the same notable properties of the rigid robot dynamic models [1], hence the control strategies used for rigid manipulators can be adopted.

As for the fast system (12), this is a marginally stable linear system that can be stabilized to the equilibrium manifold $z_f = 0$ ($\dot{z} = 0$) and $z_f = 0$ ($z = z_s$) by a proper choice of the control input $u_f$. A reasonable way to achieve this goal is to design a state-space control law of the form

$$u_f = K_1 z_f + K_2 z_f \quad (16)$$

where, in principle, the matrices $K_1$ and $K_2$ should be tuned for every configuration $\theta_s$. However, the computational burden necessary to perform this strategy can be avoided by using constant matrix gains tuned with reference to a given manipulator configuration [6]; any state-space technique can be used, e.g., based on classical pole placement algorithms.

4 Force and position regulation

The control objective consists in simultaneous regulation of the contact force $f$ to a constant set point $f_d$ and of the position $p$ to a constant set-point $p_d$.

In case of contact with an elastically compliant surface, a viable strategy is the parallel control approach [10], which is especially effective in the case of inaccurate contact modeling. The key feature is to have a force control loop working in parallel to a position control loop along each task space direction. The logical conflict between the two loops is managed by imposing dominance of the force control action over the position one, i.e., force regulation is always guaranteed at the expense of a position error along the constrained directions.

A force-position parallel regulator controller for rigid robots was proposed in [11], based on position $PD$ position control + gravity compensation + desired force feedback + $PI$ force control.

For the case of the flexible link manipulator (3), with reference to the slow system (15), the following parallel regulator can be adopted

$$u_s = \tilde{J}_\theta^T(\theta_s) k_f(p - p_s) - k_D \ddot{\theta}_s \quad (17)$$
where \( p_r \) is defined as
\[
p_r = p_d + k_F^{-1} \left( k_F (f_d - f_s) + k_I \int_0^t (f_d - f_s) \, dt \right);
\]
and \( k_F, k_P, k_D, k_I > 0 \) are suitable feedback gains.

A better insight into the behaviour of the system during the interaction can be achieved by considering a model of the compliant environment. To this purpose, a planar surface is considered, which is locally a good approximation to surfaces of regular curvature, and the model of the contact force is given by
\[
f = k_c n n^T (p - p_o)
\]
(19)
where \( p_o \) represents the position of any point on the undeformed plane, \( n \) is the unit vector along the normal to the plane, and \( k_c > 0 \) is the contact stiffness coefficient. For the purpose of this work, it is assumed that the same equation can be established in terms of the slow variables. Such a model shows that the contact force is normal to the plane, and thus a null force error can be obtained only if the desired force \( f_d \) is aligned with \( n \). Also, null position errors can be obtained only on the contact plane while the component of the position along \( n \) has to accommodate the force requirement specified by \( f_d \).

The stability analysis for the slow system (15) with the control law (17) and (18) can be carried out with the same arguments used in [11] for the case of rigid manipulators. In particular it can be shown that, if the Jacobian \( J_\theta (\theta_s) \) of the equivalent rigid manipulator is full-rank, then the closed loop system has an exponentially stable equilibrium at
\[
p_{s, \infty} = (I - n n^T) p_d + n n^T (k_c^{-1} f_d + p_o)
\]
(20)
\[
f_{s, \infty} = k_c n n^T (p_{s, \infty} - p_o) = f_d
\]
(21)
where the matrix \( (I - n n^T) \) projects the vectors on the contact plane. If \( f_d \) is not aligned with \( n \), then a drift motion of the manipulator tip is generated along the plane; for this reason, if the contact geometry is unknown, it is advisable to set \( f_d = 0 \).

As a final step, the full-order system (3) and the control law (13) with \( u_i \) in (17) and \( u_f \) in (16) have to be analyzed. By virtue of Tikhonov’s theorem it can be shown that regulation of the force \( f \) and of the position \( p \) is achieved with an order \( \epsilon \) approximation.

5 Force regulation and position tracking

The above control scheme provides regulation of the component of the tip position on the contact plane. On the other hand, if tracking of a time-varying position \( p_d(t) \) on the contact plane is desired (with an order \( \epsilon \) approximation), an inverse dynamics parallel control scheme can be adopted for the slow system, i.e.
\[
u_s = \ddot{B}_{\theta, \theta} (\theta) \ddot{J}_{\theta}^{-1} (\theta_s) \left( a_s - \ddot{h} (\theta_s, \dot{\theta}_s) \right) + \dot{c}_\theta (\theta_s, \dot{\theta}_s)
\]
(22)
where \( a_s \) is the new control input and a non-redundant manipulator has been considered. Folding (22) into (15) gives
\[
\ddot{p}_s = a_s;
\]
(23)

hence the control input \( a_s \) can be chosen as
\[
a_s = \ddot{p}_r + k_p (\ddot{p}_r - \ddot{p}_s) + k_f (p_r - p_s)
\]
(24)
where \( p_r = p_d + p_e \) and \( p_e \) is the solution of the differential equation
\[
k_a \ddot{p}_e + k_v \dot{p}_e = f_d - f_s;
\]
(25)
\( k_P, k_D, k_I > 0 \) are suitable feedback gains.

By using for the slow system the same arguments developed in [2] for rigid manipulators, it can be easily shown that the control law (22), (24),(25) ensures regulation of the contact force to the desired set-point \( f_d \) and tracking of the time-varying component of the desired position on the contact plane \((I - n n^T) p_d(t)\).

As before, Tikhonov’s theorem has to be applied to the full-order system (3) with the composite control law (13), (22), (24)-(25) and (16); it can be shown that that force regulation and position tracking are achieved with an order \( \epsilon \) approximation.

6 Simulation

In order to illustrate the effectiveness of the proposed strategy, a planar two-link flexible manipulator (Fig. 1) is considered, and an expansion with two clamped-mass assumed modes is taken for each link. The dynamic parameters can be found in [13]. The contact surface is a vertical plane, thus the normal vector in (19) is \( n = [1 \ 0]^T \); a point of the undeformed plane is \( p_o = [0.55 \ 0]^T \) m and the contact stiffness is \( k_c = 30 \text{ N/m} \).

The manipulator was initially placed with the tip in contact with the undeformed plane in the position \( p(0) = [0.55 \ -0.55]^T \) m with null contact force. It is desired to reach the tip position \( p_d = [0.55 \ -0.35]^T \) m and a 5th-order polynomial trajectory with null initial and final velocity and acceleration is imposed from the initial to the final position with a duration of 5 s. The desired force is taken from zero to the
value $f_\delta = [5 \ 0]^T N$ according to a 5th-order polynomial trajectory with null initial and final first and second derivative and a duration of 1 s. The fast control law $u_f$ has been implemented with $\epsilon = 0.1606$. The matrix gains in (16) have been tuned by solving an LQ problem for the system (12) with the configuration-dependent terms computed in the initial manipulator configuration. The matrix weights of the index performance has been chosen so that to preserve the time-scale separation between slow and fast dynamics for both the control schemes.

Simulations have been performed via MATLAB with SIMULINK. To reproduce a real situation of a continuous-time system with a digital controller, the control laws are discretized with 5 ms sampling time, while the equations of motion are integrated using a variable step Runge-Kutta method with a minimum step size of 1 ms.

In the first case study, the slow controller (17) and (18) has been used in the composite control law (13). The actual force $f$ and position $p$ are used in the slow control law instead of the corresponding slow values, assuming that direct force measurement is available and that the tip position is computed from joint angles and link deflection measurements via the direct kinematics equation (1). The control gains have been set to $k_P = 100$, $k_D = 4$, $k_f = 100$, $k_T = 500$. In Fig. 2 the time histories of the desired (dashed) and actual (solid) contact force are reported, together with the position error. It is easy to see that the contact force remains close to the desired value during the tip motion (notice that the commanded position trajectory has a 5 s duration) and reaches the desired set-point at steady state. Only the $y$-component of the desired position is regulated to the desired value, while a significant error occurs for the $x$-component; its (constant) value at steady state is exactly that required to achieve null force error along the same axis. The time histories of the joint angles and link deflections are reported in Fig. 3. It can be recognized that the oscillations of the link deflections are well damped; moreover, because of gravity and contact force, the manipulator has to bend to reach the desired tip position with the desired contact force. Fig. 4 shows the time history of the joint torque $u$ and the first 0.5 s of the time history of the fast torque $u_f$. It can be observed that the control effort keeps limited values during task execution; the control torque $u_f$ converges to zero with a transient much faster than the transient of $u$, as expected.

In the second case study, the slow controller (22), (24)-(25) has been used in the composite control law (13). As before, the actual force $f$ and position $p$ are used in the controller in lieu of the corresponding slow variables. The control gains have been set to $k_P = 100$, $k_D = 22$, $k_A = 0.7813$, $k_V = 13.75$. In Fig. 5 the time histories of the contact force and position errors are reported. This time the desired force set-point is reached after about 3 s, before completion of the tip motion; moreover, the tracking performance for the the $y$-component of the desired position is better than in the previous case study. The time histories of the joint angles and of the link deflections are reported in Fig. 6, while the time histories of the components of the joint torque vector $u$ and of the fast torque vector $u_f$ are reported in Fig. 7. It can be recognized that although the performance is better than in the previous case study, a similar control effort is required.

It is worth pointing out that the simulation of both slow control laws without the fast control action (16) has revealed an unstable behaviour; the results have not been reported for brevity.

7 Conclusion

The problem of force and position control for flexible manipulators has been considered. By using singular perturbation theory, under the assumption of large link stiffness, the system has been split into a slow subsystem describing the rigid dynamics and a fast subsystem describing the flexible dynamics. Then a force and position parallel control has been adopted for the slow subsystem, while a fast action has been designed for vibration damping. A simulation study has confirmed the feasibility of the proposed approach.

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References


Figure 2: Time histories of contact force and position error in the first case study.

Figure 5: Time histories of contact force and position error in the second case study.

Figure 3: Time histories of joint angles and link deflections in the first case study.

Figure 6: Time histories of joint angles and link deflections in the second case study.

Figure 4: Time histories of joint torques and fast control in the first case study.

Figure 7: Time histories of joint torques and fast control in the second case study.


