Identifiable Parameters for Parallel Robots Kinematic Calibration

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Abstract

This paper presents a numerical method for the determination of the identifiable parameters of parallel robots. The special case of Stewart-Gough 6 degrees-of-freedom parallel robots is studied for classical and self calibration methods, but this method can be generalized to any kind of parallel robot. The method is based on QR decomposition of the observation matrix of the calibration system. Numerical relations between the parameters which are identified and those which are not identifiable can be obtained for each method.

1. Introduction

The aim of the kinematic calibration is to calculate accurately the values of the kinematic parameters of the robot in order to improve its accuracy.

The classical methods for parallel robot calibration need external sensors to measure the position and orientation of the mobile platform [1] [2] [3] [4] [5]. The calibration problem is formulated in terms of minimizing the difference between the measured and computed motorized joint variables, it uses the inverse kinematic model that is easy to calculate for parallel robots. Self calibration methods using extra sensors on the passive joints have been also proposed for parallel robots [6] [7] [8] [9]. These calibration methods are based on the use of redundant sensors on the passive joints adjust the values of the kinematic parameters in order to minimize a residual between the measured and the calculated values of the angles of these joints. As many parallel robots don’t have redundant sensors on the passive joint, mechanical constraints on the leg can also be used to calibrate the robot [10] [11].

It is well known that for some calibration methods, all the geometric parameters cannot be identified: some of them have no effect on the calibration model, and some other are grouped together. In previous work, the identifiable parameters of parallel robots are derived by intuition. In the case of serial robots, the identifiable parameters are computed from a QR decomposition of the analytical observation matrix [12]. We propose to extend this method for parallel robots even in the case where the identification Jacobian matrix cannot be obtained analytically.

2. Description of the robot

Figure 1: Stewart-Gough parallel robot

The parallel robot studied here is composed of a fixed base and a movable platform connected with six legs of motorized variable length (Figure 1). The base connections are composed of Universal joints (U-joints) and the platform connections are composed of Spherical joints (S-joints). The centers of the U-joints and S-joints are respectively denoted by Ai and Bi (i = 1 to 6). We suppose that the U-joints are composed of two revolute and intersecting joints, while the S-joints are composed of three intersecting revolute joints. The configuration of the parallel robot is given by the (6x1) vector $L$ representing the leg lengths $A_iB_i$ for $i=1,...,6$. 
\[ L = [ l_1 \ l_2 \ l_3 \ l_4 \ l_5 \ l_6 ]^T \tag{1-a} \]

Typically each variable is given as:
\[ l_i = q_i + q_{off,i} \tag{1-b} \]

where \( q_i \) is the prismatic position sensor reading and \( q_{off,i} \) is a fixed offset value.

### 2.1 The geometric parameters

![Figure 2: Definition of the frames](image)

We define the frame \( F_0 \) fixed with respect to the base and the frame \( F_m \) fixed with respect to the movable platform. They are defined as follow [7]:

- \( A_1 \) is the origin of frame \( F_0 \), while the \( x_0 \) axis is determined by \((A_1A_2)\) and \( x_0y_0 \) plane is determined by the points \( A_1, A_2 \) and \( A_6 \).

- similarly, \( B_1 \) is the origin of frame \( F_m \), while \( B_1B_2 \) represents its \( x_m \) axis and \( B_1B_2B_6 \) its \( x_my_m \) plane.

With this definition of \( F_0 \) and \( F_m \) we have:
\[ ^0P_{A1} = 0^0P_{YA1} = 0^0P_{ZA1} = 0^0P_{ZA2} = 0^0P_{ZM6} = 0 \]
\[ ^mP_{B1} = ^mP_{YB1} = ^mP_{ZB1} = ^mP_{YB2} = ^mP_{ZB2} = ^mP_{ZB6} = 0 \]

where \( ^jP_i \) denotes the coordinates of the point \( P_i \) with respect to coordinate system \( F_j \) and:
\[ ^jP_i = [ ^jP_{x_i} \ ^jP_{y_i} \ ^jP_{z_i} ]^T \]

Thus, the robot is described by 24 constant coordinates that may be not equal to zero.

The \((4x4)\) transformation matrix between frames \( F_0 \) and \( F_m \) giving the location (position and orientation) of the platform with respect to the base is denoted by:
\[ ^0T_m = \begin{bmatrix} 0^0A_m & 0^0P_m \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} \]

This matrix is a function of the 24 parameters and the 6 leg lengths of the robot.

The location of the base frame \( F_0 \) with respect to the world reference frame \( F_1 \) of the environment is given by a transformation matrix \( Z \). In addition, the matrix \( E \) denotes the location of the end-effector frame \( F_E \) in frame \( F_m \) of the platform (cf. Figure 2). The location of the end-effector frame relative to the world reference frame is:
\[ ^{-1}T_E = Z^0T_m \cdot E \tag{3} \]

Thus, the coordinates of point \( A_i \) relative to frame \( F_1 \) are given by:
\[ \begin{bmatrix} ^{-1}P_{A_i} \\ 1 \end{bmatrix} = ^{-1}T_0 \cdot \begin{bmatrix} 0^0P_{A_i} \\ 1 \end{bmatrix} = Z \cdot \begin{bmatrix} 0^0P_{A_i} \\ 1 \end{bmatrix} \tag{4} \]

The coordinates of point \( B_i \) relative to frame \( F_E \) are:
\[ \begin{bmatrix} ^E P_{B_i} \\ 1 \end{bmatrix} = ^E T_m \cdot \begin{bmatrix} m^mP_{B_i} \\ 1 \end{bmatrix} = E \cdot \begin{bmatrix} m^mP_{B_i} \\ 1 \end{bmatrix} \tag{5} \]

As the matrices \( Z \) and \( E \) can be defined arbitrarily by the user as a function of the task, 6 independent parameters are necessary to define each of them. We can use for example 3 position parameters and 3 Euler angles.

To conclude, we can describe the geometry of the robot using 36 constant parameters: either by \( ^{-1}P_{A_i} \) and \( ^E P_{B_i} \), or by \( 0^0P_{A_i}, \ m^mP_{B_i} \) and the matrices \( Z \) and \( E \). The total number of parameters is thus equal to 42, after taking into account the 6 joint variables. These parameters permit to calculate the direct and inverse kinematic models.

For the calibration, we propose to use directly the coordinates of points \( A_i \) and \( B_i \) in frames \( F_1 \) and \( F_E \) respectively in order to have homogeneous parameters to identify (only lengths). When the identification is done, it is easy to find the transformations \( Z \) and \( E \), and the coordinates of the points of the base and the platform in frames \( F_0 \) and \( F_m \).

### 2.2 Kinematic modeling

The inverse kinematic model (IKM) which computes the leg lengths vector for a desired \( ^{-1}T_E \) is unique and easy to
obtain [13]. While, the direct kinematic model (DKM), which gives the matrix $^{-1}T_h$ as a function of a given leg lengths vector, is difficult to obtain analytically and up to 40 solutions may exist [14]. A numerical iterative method based on the inverse Jacobian matrix is used to find a local solution for the DKM. Such procedure converges quickly.

3. General calibration models

The aim of the kinematic calibration is to estimate accurately the geometric parameters. All the calibration methods are based on calculating a function, for sufficient number of configurations, in terms of the robot parameters and some external variables. The model parameters are estimated by minimizing this function by solving a nonlinear system of equations.

The general form of the calibration equation is:

$$F(Q, X, \eta_i) = \begin{bmatrix} f_1(q^1, x^1, \eta_i) \\ \vdots \\ f_e(q^e, x^e, \eta_i) \end{bmatrix} = 0$$  \hspace{1cm} (6)

where $\eta_i$ denotes the real (unknown) geometric parameters, $Q = \{ q^1, \ldots, q^e \}^T$ contains the prismatic positions of the robot for $e$ different configurations, and $X = \{ x^1, \ldots, x^e \}^T$ are the corresponding external measured variables such as the Cartesian coordinates.

This nonlinear optimization problem can be solved by the `leastsq` function of Matlab based on the Levenberg-Marquardt method.

Supposing that U-joints and S-joints are perfect, we have to identify the error $\Delta^1P_{A1}, \Delta^2P_{B1}, \Delta d_{off,i}$ (with $i = 1, \ldots, 6$).

The nominal parameters of the robot are denoted by the vector $\eta$ while $\Delta \eta$ is the error vector (thus we have $\eta_e = \eta + \Delta \eta$). In the general case there are 42 parameters to identify, but they are not all identifiable with every calibration method. Before solving the calibration equation, it is important to define the identifiable parameters, because only these parameters can be identified without ambiguity. We propose to determine these parameters using QR decomposition of the observation matrix of the linearized model of randomly $e$ configurations satisfying the constraints of the calibration procedure. The outlines of this algorithm is given in Appendix A. The linearized system of equation corresponding to the nonlinear equation (6) can be written as:

$$\Delta Y(Q, X, \eta) = W(Q, \eta) \cdot \Delta \eta + \rho$$  \hspace{1cm} (7)

where $\Delta Y$ is the difference between the model and the real robot, $W$ is the $(r \times np)$ observation matrix of the system, with $np$ the number of geometric parameters and $r >> np$.

The vector $\rho$ indicates the residual errors owing to noise or modeling errors because of non geometric parameters such as: thermal effect, manufacturing errors in the U or S joints, ...

For parallel robots, the observation matrix $W$ can be obtained analytically for the calibration method which is based on the IKM (see section 4.1). For all the other methods we have to calculate $W$ numerically by supposing small variations $\epsilon$ on each geometric parameter and calculating the corresponding $\Delta Y_i$. The $j$th column of $W$ corresponding to that parameter will be computed as $\Delta Y_j / \epsilon$. Good results are obtained taking $\epsilon = 10^{-6}$ meter for each parameter.

The number of the identifiable parameters denoted by $b$ is fixed as a function of the calibration method, but the set of the identifiable parameters is not unique. The QR decomposition will provide as a set of identifiable parameters those corresponding to the first $b$ independent columns of $W$. We assign a priority number to each parameter, the parameters with higher priority will be placed at first in $\eta$.

We place at first the offsets $q_{off,i}$ (priority 3), and we place at the end the 12 coordinates of the points defining frames $F_0$ and $F_m$ (i.e. $P_{X1}, -1P_{Y1}, -1P_{Z1}, -1P_{A2}, -1P_{Z2}, -1P_{A6}$, $P_{Xb1}, P_{Yb1}, P_{Zb1}, P_{Xb2}, P_{Yb2}$, $P_{Zb6}$) (priority 1), the other parameters will get priority 2 and will be placed after the offset parameters in the following order:

$P_{X1}$, $P_{Y1}$, $P_{Z1}$, $P_{A2}$, $P_{Z2}$, $P_{A6}$, $P_{Xb1}$, $P_{Yb1}$, $P_{Zb1}$, $P_{Xb2}$, $P_{Yb2}$, $P_{Zb6}$.

4. Application to calibration methods

We compute the identifiable parameters for several calibration methods for the parallel robot whose nominal parameters are given in Table 1. The obtained identifiable parameters are valid for any robot of the Stewart-Gough type. The grouping relations of the non identifiable parameters are functions of the numerical values of the geometric parameters.
The equation to solve for each leg and each configuration is:

\[
\Delta q_i = \frac{1}{L_i} \begin{bmatrix}
\Delta^{-1}P_{A_i} - A_E^{-1} \Delta E P_{B_i} - P_{E_i}^{-1} \\
\Delta^{-1}A_E^{-1} \Delta E P_{B_i} - \Delta E P_{E_i}^{-1} \\
\Delta E P_{E_i}^{-1}
\end{bmatrix}^T \begin{bmatrix}
\Delta^{-1}P_{A_i} \\
\Delta^{-1}A_E^{-1} \\
\Delta E P_{E_i}^{-1}
\end{bmatrix}
\]

Applying this equation for the 6 legs of the robot and e configurations, we have the relation:

\[
\Delta Q = W(\mathbf{X},\mathbf{\eta}) \Delta \mathbf{\eta}
\]

where \(\Delta Q = \mathbf{Q}' - \mathbf{Q}\) is the difference between the measured prismatic joint values \(\mathbf{Q}'\) and those computed by the IKM \(\mathbf{Q}\).

In this case, the observation matrix \(W\) is defined analytically. If we write the system of equations (9) for e random configurations, with \(e \gg 7\) such that the number of rows of \(W\) is greater than the number of the parameters. The rank of the matrix \(W\) is 42. Thus, all the geometric parameters can be identified.

The condition number of \(W\) is obtained as 39. The identifiable parameters of the system are obtained by the QR decomposition of this matrix. Applying the rules of priority described in section 3, the errors \(\Delta^E P_{Y_{B2}}, \Delta^E P_{Z_{B2}}\) and \(\Delta^E P_{Z_{B6}}\) are not identifiable and their effect are grouped on the other parameters which defines the positions of the S-joints on the mobile platform. We propose to fixe these parameters such that:

\[
\begin{align*}
E P_{Y_{B2}} &= E P_{Z_{B2}} = E P_{Z_{B6}} = 0
\end{align*}
\]

4.2 Calibration with measurement of the position of the platform

Measuring only the position of the platform, we cannot use the IKM of the robot since we have only 3 equations to solve a system of 6 unknowns (the 6 leg lengths of the robot). Nevertheless, using the direct kinematic model (DKM), if we consider a configuration \(q\) of the robot and \(P_0\) the measured position of the effector in frame \(F_1\), we can write the nonlinear model of calibration as:

\[
-\mathbf{p}_E(q,\mathbf{\eta}) - \mathbf{p}_E = 0
\]

The linear differential model which take into account the errors on the geometric parameters is:

\[
\Delta \mathbf{p}_E = \Psi(q,\mathbf{\eta}) \Delta \mathbf{\eta}
\]

The Jacobian matrix \(\Psi\) is obtained numerically by supposing small variation on each parameter and calculating the corresponding variation on \(\Delta \mathbf{p}_E\) (cf. section 3).

Measuring the position of the end-effector for a sufficient number \(e\) of random configurations (minimum 14 configurations), we have:

\[
\begin{bmatrix}
\Delta \mathbf{p}_E^1 \\
\vdots \\
\Delta \mathbf{p}_E^e
\end{bmatrix} = \begin{bmatrix}
\Psi^1(q',\mathbf{\eta}) \\
\vdots \\
\Psi^e(q',\mathbf{\eta})
\end{bmatrix} \Delta \mathbf{\eta} = W(q,\mathbf{\eta}) \Delta \mathbf{\eta}
\]

The rank of the matrix \(W\) is obtained as 39. The identifiable parameters of the system are obtained by the QR decomposition of this matrix. Applying the rules of priority described in section 3, the errors \(\Delta^E P_{Y_{B2}}, \Delta^E P_{Z_{B2}}\) and \(\Delta^E P_{Z_{B6}}\) are not identifiable and their effect are grouped on the other parameters which defines the positions of the S-joints on the mobile platform. We propose to fixed these parameters such that:

\[
\begin{align*}
E P_{Y_{B2}} &= E P_{Z_{B2}} = E P_{Z_{B6}} = 0
\end{align*}
\]
This convention makes that the orientation of frame E, which cannot be determined, is such that the x axis is along the measured point and the point B2 while the xy plane is along the measured point and the points B2 and B6. The condition number of the linear observation matrix W for this calibration method using a number of equations which is equal to 4 times the number of parameters is about 2000.

4.3 Calibration using two inclinometers

This calibration method is similar to the previous one: a partial measurement is done on the orientation coordinates of the platform. The rotation angles of the platform of the robot about x_m and y_m axis are measured by two inclinometers fixed on the platform [5].

For a given configuration q, the theoretical values $\alpha_i$ and $\alpha_e$ of the inclinometers can be computed using the DKM. These values are function of some elements of orientation matrix $^{-1}A_e$ and a supplementary angle parameter $\gamma$ which defines the orientation between the two inclinometers axes. The angle $\gamma$ is not always well known and must be added to the parameters to identify.

The following linear differential model can be written:

$$\Delta \Phi = \Psi(q, \eta, \gamma) \cdot \Delta \eta$$

where $\Delta \Phi$ is the difference vector between the inclinometers measured values $\Phi'$ and those computed by the model $\Phi$, and $\Psi$ is the numerical Jacobian matrix (cf. section 3).

With a sufficient number e of configurations:

$$\left\{ \begin{array}{c}
\Delta \Phi^1 \\
\Delta \Phi^e
\end{array} \right\} = W(Q, \eta, \gamma) \cdot \Delta \eta$$

The rank of W is 36 and the 7 non identifiable parameters are given by the QR decomposition of the observation matrix. There are 4 non identifiable parameters on the position of the U-joints ($\Delta^1P_{X_{A1}}, \Delta^1P_{Y_{A1}}, \Delta^1P_{Z_{A1}}$ and $\Delta^1P_{Y_{A2}}$) and 3 on the position of the S-joints ($\Delta^S_P_{X_{B1}}, \Delta^S_P_{Y_{B1}}$ and $\Delta^S_P_{Z_{B1}}$). The effect of these parameters are respectively grouped on the other parameters of the base (U-joints) and the platform (S-joints).

These results are confirmed by the study of the geometry of the system. The position coordinates of the inclinometers on the platform have no effect on the values they give. Consequently we can consider that the origin $O_e$, which cannot be determined by this method, is aligned with the origin of frame $F_m$. Then we have by convention:

$$E_{P_{X_{B1}}} = E_{P_{Y_{B1}}} = E_{P_{Z_{B1}}} = 0$$

Similarly, the position of the base of the robot with respect to $F_{-1}$ has no influence on the inclinometers measurement, as well as its orientation around the vertical axis. We can define arbitrarily the origin of frame $F_{-1}$ as $A_1$ and the axis $x_{-1}$ and $z_{-1}$ such that $A_2$ is in the plane ($A_1x_{-1}z_{-1}$). Then we have by definition:

$$^{-1}P_{X_{A_1}} = ^{-1}P_{Y_{A_1}} = ^{-1}P_{Z_{A_1}} = ^{-1}P_{Y_{A_2}} = 0$$

The condition number of the linear observation matrix W for this calibration method using a number of equations which is equal to 4 times the number of parameters is about 2000.

4.4 Calibration with mechanical constraints on the orientation of the legs

This calibration method uses only the variables of the motorized prismatic joints corresponding to configurations where either one U-joint or one S-joint is fixed by mechanical lock, thus the corresponding leg direction is constant with respect to the fixed base or with the movable platform [11].

Each U-joint $i$ is described by 2 angles $q_{1,i}$ and $q_{2,i}$, while each S-joint $i$ is defined using three angles $q_{3,i}, q_{4,i}$ and $q_{5,i}$. For a given configuration $q$, these angles can be computed for every legs by a generalized direct kinematic model [11]:

$$h_{u_{ij}} = \begin{cases} 
q_{1,i} \\
q_{2,i}
\end{cases} = g(q, \eta)$$

$$h_{s_{ij}} = \begin{cases} 
q_{3,i} \\
q_{4,i} \;
q_{5,i}
\end{cases} = h(q, \eta)$$

4.4.1 Fixing the U-joint of a leg

Supposing 2 configurations $q^a$ and $q^b$ for which the $i^{th}$ U-joint has been locked. The nonlinear error function between them is given as:

$$y_{u_{ij}}^a(q^a, \eta) - y_{u_{ij}}^b(q^b, \eta) = 0$$
The differential system of equation is given as:
\[
\Delta y_{u_i} = y_{u_i}^a - y_{u_i}^b = \Psi(q, \eta) \cdot \Delta \eta
\] (20)

With a sufficient number \( e \) of configurations:
\[
C_{u_i} = \begin{bmatrix}
\Delta y_{u_i}^1 \\
\vdots \\
\Delta y_{u_i}^{e-1} \\
\end{bmatrix} = \begin{bmatrix}
y_{u_i}^1 - y_{u_i}^2 \\
\vdots \\
y_{u_i}^{e-1} - y_{u_i}^e \\
\end{bmatrix} = W(Q, \eta) \cdot \Delta \eta
\] (21)

where \( W \) is computed numerically.

The rank of \( W \) is 29, giving 7 non identifiable parameters of the base and 6 non identifiable parameters for the movable platform. The interpretation of this supplementary non identifiable parameter is given in section 4.4.4.

The condition number of the linear observation matrix \( W \) for this calibration method using a number of equations which is equal to 4 times the number of parameters is about 700.

### 4.4.2 Fixing the S-joint of a leg

Supposing a set of configurations \( Q = \{ q^1, \ldots, q^e \} \) for which the \( i^{th} \) S-joint has been locked, we can write a differential calibration system of \((3x(e-1))\) linear equations by the use of relation (19):
\[
C_{s_i} = \begin{bmatrix}
\Delta y_{s_i}^1 \\
\vdots \\
\Delta y_{s_i}^{e-1} \\
\end{bmatrix} = \begin{bmatrix}
y_{s_i}^1 - y_{s_i}^2 \\
\vdots \\
y_{s_i}^{e-1} - y_{s_i}^e \\
\end{bmatrix} = W(Q, \eta) \cdot \Delta \eta
\] (22)

where \( W \) is computed numerically.

The rank of \( W \) is 29, this gives 6 non identifiable parameters for the base and 7 non identifiable parameters for the platform.

The condition number of the linear observation matrix \( W \) for this calibration method using a number of equations which is equal to 4 times the number of parameters is about 7500.

### 4.4.3 Mixing the data of locking different joints

If two or more sets of configurations are used, where in each set either an U-joint or a S-joint has been fixed, the rank of the numerical observation matrix \( W \) of the calibration system is 30. The \( QR \) decomposition of this matrix shows that 6 parameters of the base and 6 parameters of the platform are not identifiable. With the priority rules, defined in section 3, these parameters correspond to those which define the base and the end-effector transformation matrices \( Z \) and \( E \). In practice we put them equal to zero:
\[
-1p_{x_{A_1}} = -1p_{y_{A_1}} = -1p_{z_{A_1}} = -1p_{z_{A_2}} = -1p_{z_{A_2}} = 0
\] (23)
\[
-1p_{x_{B_1}} = -1p_{y_{B_1}} = -1p_{z_{B_1}} = -1p_{z_{B_2}} = -1p_{z_{B_2}} = 0
\] (24)

This gives: frame \( F_i = F_0 \) and \( F_m = F_E \).

The condition number of the linear observation matrix \( W \) for this calibration method using a number of equations which is equal to 4 times the number of parameters is about (with one U-joint and one S-joint locked) using four time the number of equations that are necessary is about 700.

### 4.4.4 Comments

This calibration method is an autonomous one and it is easy to see that \( Z \) and \( E \) transformations matrices have no effect on the computation of the angles of each leg with respect to the base or the platform. Then, with the parameterization we have chosen, the maximum number of identifiable parameters by an autonomous calibration method is 30.

When only one set of configurations is used with one U-joint (respectively one S-joint) locked, a \( 7^{th} \) geometric parameter of the base (respectively of the platform) cannot be identified. If we analyze the numerical relations between this non identifiable parameter and the identifiable ones, we observe a dependence between the offset \( q_{off} \) of the locked leg and the position of the center of the U-joint (respectively the S-joint) of this leg along the axis of its prismatic motorized joint. Thus, placing the center of the locked joint along its leg direction will satisfy the locking constraint (cf. figure 3). That is why we have a non identifiable parameter more. This situation has not been discovered in reference [11], but it has been shown that two different joints must be locked to get good results.

### 4.5 Calibration with extra sensors on some passive Universal joints

Zhuang [9] has presented autonomous methods based on the use of extra sensors on some passive U-joints. Knowing a set of \( e \) random configuration \( Q \) and the real (measured) values of \( \theta_{u_j} \) and \( \theta_{s_j} \) of U-joint \( j \) for each configuration, the following linear differential system can be written from (18):

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\begin{equation}
\begin{pmatrix}
(y^1_{u_i})^T - y_{u_i}^1 \\
\vdots \\
(y^r_{u_i})^T - y_{u_i}^r
\end{pmatrix} = W(Q, \eta) \cdot \Delta \eta
\end{equation}

where \( \left( y^i_{u_i} \right)^T \) is the vector of the values measured for the \( i^{th} \) U-joint and \( y_{u_i}^1 \) is computed from the generalized DKM.

Figure 3: Two different robots with the same orientation of one leg for a given configuration \( q \)

Results are given in \cite{9} with 6 extra sensors on 3 passive U-joint. The condition number of the linear observation matrix \( W \) for this calibration method (with 6 extra sensors) using a number of equations which is equal to 4 times the number of parameters is about 350.

The QR decomposition of \( W \) shows that using only one sensor reading is sufficient to identify 30 parameters which is the maximum for a self calibration method. In this case, the condition number of \( W \) is about 1500. This means that the increase of the number of measured angles increases the observability of the system.

5. Conclusion

This paper presents a generalized method which gives the identifiable and non identifiable geometric parameters for the calibration methods of parallel robots. This method is based on the QR decomposition of a numerical observation matrix of the calibration system which is obtained numerically by supposing small variations on each geometric parameter of the model. Results are given for several methods, the physical interpretation of the non identifiable parameters has been given. The observability measure of each method is given by the condition number of the observation matrix of the linearized model.

References


Appendix A: Calculation of the identifiable parameters

Let us consider the following overconstrained system of linear equations:

\[ \Delta Y = W \Delta \eta \]  
(a-1)

where \( W \) is an \((r \times c)\) matrix with \(r >> c\). If \( b \) is the rank of this system it can be rewritten as:

\[ \Delta Y = \begin{bmatrix} W_1 & W_2 \end{bmatrix} \begin{bmatrix} \Delta \eta_1 \\ \Delta \eta_2 \end{bmatrix} \]  
(a-2)

\[ W_2 = W_1 \beta \]  
(a-3)

where \( W_1 \) represents \( b \) independent columns of \( W \) and \( W_2 \) represents \((c-b)\) dependent columns of \( W \), and \( \beta \) is a \((b \times (c-b))\) matrix with constant elements.

Using (a-3), equation (a-2) can be written as:

\[ \Delta Y = W_1 \Delta \eta_b \]  
(a-4)

with

\[ \Delta \eta_b = \Delta \eta_1 + \beta \Delta \eta_2 \]  
(a-5)

In the identification process, equation (a-4) will be used instead of (a-1). The solution will directly yield \( \Delta \eta_b \), which is called the identifiable parameters vector. The matrix \( \beta \) is not needed in the identification process.

Numerically, the study of the identifiable parameters is equivalent to study the space spanned by the columns of \((r \times c)\) matrix \( W \) with \(r >> c\), calculated from the differential model using a sufficient number of configurations satisfying the constraint of the problem.

The QR decomposition of matrix \( W \) is given as [15] [16]:

\[ Q^T W = \begin{bmatrix} R \\ 0_{(c-b) \times c} \end{bmatrix} \]  
(a-6)

where \( Q \) is an \((r \times r)\) orthogonal matrix, \( R \) is a \((c \times c)\) upper triangular matrix, and \(0_{i \times j} \) is the \((i \times j)\) zero matrix.

Theoretically, the non identifiable parameters are those whose corresponding elements on the diagonal of the matrix \( R \) are equal to zero. Thus, assuming \( \tau \) is the numerical zero, if the element \( |R_{ii}| = \tau \), the corresponding parameter \( \Delta \eta_i \) is not identifiable. The numerical zero \( \tau \) can be taken as [15]:

\[ \tau = c \cdot \epsilon \cdot \max |R_{ii}| \]  
(a-7)

where \( \epsilon \) is the machine precision.

To calculate the identifiable (base) parameters as function of the standard parameters, let us permute the columns of \( W \) such that the first \( b \) columns are independent:

\[ W P = \begin{bmatrix} W_1 & W_2 \end{bmatrix} \]  
(a-8)

where \( P \) is a permutation matrix, \( W_1 \) represents the \( b \) independent columns of \( W \), and \( W_2 \) represents the \((c-b)\) dependent columns of \( W \).

The QR decomposition of \((W P)\) gives:

\[ [W_1 \ W_2] = [Q_1 \ Q_2] \begin{bmatrix} R_1 \ R \\ 0 \ 0 \end{bmatrix} \]  
(a-9)

where \( R_1 \) is a \((b \times b)\) regular matrix. Then it comes:

\[ W_2 = W_1 R_1^{-1} R_2 \]  
(a-10)

thus from (a-3) and (a-9) we get:

\[ \beta = R_1^{-1} R_2 \]  
(a-11)