Adaptive Control of a Flexible Joint Manipulator

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Abstract

In this paper control of a flexible joint manipulator is designed using an adaptive output-feedback controller based on a backstepping design. The parameters of system are assumed to be unknown and only motor position and link position measurements are used for the synthesis of the controller. A canonical state representation of the system is derived and filters are designed to obtain the estimates of the derivatives of motor angle and link angle. Then adaptive control law for the trajectory control of link displacement angle is derived. Compared to the same adaptive controller which measures just a link displacement angle, the order of the proposed controller becomes a half (order of 2) and complexity of control law can be reduced dramatically. Also the proposed controller does not require the bounds on uncertain parameters for the control law derivation compared other output adaptive control law.

1 Introduction

It has been a usual procedure to model the manipulator by chains of rigid links with rigid transmission joints to simplify dynamic analysis and subsequent controller design. However, the effect of link and joint flexibility are often observed in practice and recently many researchers are working on the design of controllers which include effects of joint and link flexibility. Specially when the robot is driven by actuators with high gear ratios, such as harmonic drives, joint flexibility needs to be considered for controller design. In this paper nonlinear model based output adaptive controller is designed for the rigid-link flexible-joint manipulator.

A number of researchers have proposed feedback linearization scheme, robust, and nonlinear controllers for the flexible joint manipulator. [1-4] However these schemes need either exact knowledge of the robot parameters or measurements of link position, link velocity, actuator position, and actuator velocity. Even if robot parameters are known, measuring all states of the system is not practical for actual implementation of the controllers. Some [5] considered the observer design to which requires only link position and velocity for asymptotic link position tracking.

In this paper, an adaptive controller is derived for a rigid-link flexible-joint robot which does not need any knowledge of robot parameters and only motor and link position measurement is used for asymptotic tracking of link position. It should be noted that the measurement of motor position reduces the order of the system from four to two, which simplifies the derivation of backstepping based adaptive control law dramatically. In this approach, a canonical state variable representation of the system is derived by reconstructing original state variables. Based on this new state variable form of a rigid-link and flexible-joint robot system, filters are designed and an estimate of states are constructed using a linear combination of the states of the filters. Then based on the backstepping design technique of Kanellakopoulos et al. [7] and Krsiti et al. [8], an adaptive control law for the control of link position is derived. Compared to the variable structure adaptive controller of Ref. [6] which uses only output feedback, it is assumed that the bounds on uncertain parameters are not needed for the control law derivation. Furthermore, this controller uses a high gain feedback which often leads to control saturation and may cause instability. Simulation results are presented to show the adaptive tracking capability of the link displacement of the control system.

2 Dynamic Model

Consider the single link robotic manipulator whose joint is modeled as a linear torsional spring as shown in Fig.1. Let \( q_1 \) and \( q_2 \) be the angular position of the link and motor shaft respectively, the following dynamic equations of motion are obtained:

\[
J_1 \ddot{q}_1 + F_1 \dot{q}_1 + K(q_1 - \frac{q_2}{N}) + mgd \sin(q_1) = 0
\]

(1)

\[
J_2 \ddot{q}_2 + F_2 \dot{q}_2 - \frac{K}{N}(q_1 - \frac{q_2}{N}) = u
\]

(2)

where \( J_1, J_2 \) are inertias of the link and motor, \( F_1, F_2 \) are viscous frictional constants, \( K \) is the spring constant of the joint stiffness, \( m \) is the link mass, \( d \) is the location of mass center and \( g \) is for gravity. \( N \) is the
gear ratio of the drive train and $u$ is the input torque to the system.

![Figure 1: Single link flexible joint robot](image)

Since we assume that both motor and link positions are measurable, the order of the system is reduced from four to two and one obtains a state variable representation of Eq. (1) and (2) in the following form:

$$
q = \begin{bmatrix}
0 & 0 & 1 & 0 \\
-J_1 \frac{k}{J} & 0 & 0 & 1 \\
J_1 \frac{k}{N J} & -J_1 \frac{k}{k} & 0 & 0 \\
N^2 J_2 & 0 & 0 & -F_2 \\
\end{bmatrix}
q + 
\begin{bmatrix}
0 \\
0 \\
\frac{mgd}{J_1} \\
- \frac{1}{J_2} \\
\end{bmatrix}
\sin(q) + 
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
\end{bmatrix}
u
$$

(3)

where $q = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$ and it also can be written as

$$
\dot{q} = \begin{bmatrix}
I_{2x2} & 0_{2x2} \\
M_1 & M_2 \\
\end{bmatrix}q + 
\begin{bmatrix}
0 \\
G \\
\end{bmatrix}\sin(q) + 
\begin{bmatrix}
0 \\
B \\
\end{bmatrix}u
$$

(4)

where $M_1$, $M_2$, $G = (g_1, g_2)^T$, and $B = (b_1, b_2)^T$ are appropriate matrices which can be determined from (3). Here we are interested in deriving adaptive control system so that $q_1$ tracks $q$, generated by the second order command generator asymptotically, and in the closed-loop system the state vector $q$ converges to zero as $t \to \infty$.

3 State Transformation and Filters

Since $\dot{q}_1$ and $\dot{q}_2$ are not measured, it is necessary to obtain an estimate of these variables for controller synthesis. It is useful to get a canonical form of the system to design certain filters and a linear combination of filter states provide an estimate of the state vector. Consider the following state transformation:

$$
x = \begin{bmatrix}
I_{2x2} & 0_{2x2} \\
-M_2 & I_{2x2} \\
\end{bmatrix}q
$$

(5)

where $x = (x_1, x_2, x_3, x_4)^T$, $x_1 = q_1$, and $x_2 = q_2$. Then a new state variable representation of (1) and (2) becomes

$$
\dot{x} = \begin{bmatrix}
M_2 & 0 \\
0 & M_1 \\
\end{bmatrix}x + 
\begin{bmatrix}
M_2 \\
M_1 \\
\end{bmatrix}\frac{x}{x} + 
\begin{bmatrix}
0 \\
G \\
\end{bmatrix}\sin(x) + 
\begin{bmatrix}
0 \\
B \\
\end{bmatrix}u
$$

(6)

Eq. (6) can be modified in the following form by noting that a vector $\theta$ is made of unknown parameters of the system to be controlled.

$$
\dot{x} = \begin{bmatrix}
M_2 & 0 \\
0 & M_1 \\
\end{bmatrix}x + 
\begin{bmatrix}
M_2 & 0 \\
0 & M_1 \\
\end{bmatrix}\frac{x}{x} + 
\begin{bmatrix}
B \\
M_{201}^T \\
M_{202}^T \\
M_{301}^T \\
G \\
\end{bmatrix} \begin{bmatrix}
\theta \\
\end{bmatrix}
$$

(7)

where $M_{ik}$ denotes the $k$-th row of $M_i$ and $F^T(x_1, x_2, u)$ is a 4x12 matrix. Using the definition of matrix $F$, (7) can be written as

$$
\dot{x} = Ax + Lx + F^T(x_1, x_2, u)\theta
$$

(8)

where

$$
A = \begin{bmatrix}
-L_1 & I \\
-L_2 & 0 \\
\end{bmatrix} \\
L_1 = \begin{bmatrix}
L_1_{11} & 0 \\
L_1_{21} & 0 \\
\end{bmatrix} \\
L_2 = \begin{bmatrix}
L_2_{11} & 0 \\
L_2_{21} & 0 \\
\end{bmatrix}
$$

The matrices $L_1$, $L_2$ are chosen so that $A$ is a stable matrix and satisfy the following Lyapunov equation.
\[ PA + A^T P = -I \quad \text{for} \quad P^T = P > 0 \quad (9) \]

Eq. (8) is a canonical representation of the system in which a matrix \( A \) is in a special form and the regressor matrix \( F \) is a function of the measured variables \((x_i, x_2)\) and the input, \( u \), and all the unknown parameters of the system is included in the vector \( \theta \).

Consider the filters given by [8]

\[
\begin{align*}
\dot{\xi} &= A\xi + Lux \\
\dot{\Omega}^T &= A\Omega^T + F^T(x_i, x_2, u)^T
\end{align*}
\]

where \( \xi \in \mathbb{R}^4 \) and \( \Omega^T \in \mathbb{R}^{4+2} \). For simplicity in the filter design one can rewrite the \( \Omega \) filter using the special form of the matrix \( F \) as

\[
\Omega^T = [v_1, v_2, s_2, \ldots, s_m] \equiv [v_1, v_2, S] \quad (11)
\]

Define the state estimate as

\[
\hat{x} = \xi + \Omega^T \theta \quad (12)
\]

where \( \hat{x} = x - \bar{x} \) and \( \bar{x} \) is the state error. From (9)-(12) the state error \( \bar{x} \) is governed by

\[
\dot{\bar{x}} = A\bar{x} \quad (13)
\]

where \( A \) is a Hurwitz matrix. Since \( \bar{x}(t) \rightarrow 0 \) as \( t \rightarrow \infty \), \( \bar{x}(t) \) will converge to \( x(t) \) asymptotically. Of course, (12) cannot be used to reconstruct \( \hat{x}(t) \) since \( \theta \) is unknown, however it will be used in the derivation of an adaptive control law.

### 4 Adaptive Control Laws

Let \( y=q_1 \) be a smooth trajectory which is to be tracked by \( q_1 \). From (6) and (12), the derivative of the output \( q_1 \) becomes

\[
\begin{align*}
\dot{q}_1 &= x_1 + M_{201} [q_1, q_2]^T \\
&= \xi^* + \Omega_{\theta}^T \theta + \bar{x} + M_{201} [q_1, q_2]^T \\
&= \xi^* + \bar{x} + b_1 v_{13} + [\ell v_{13}, S_{13} + e_i^T q_i + e_i^T q_3] \theta \\
&= \xi^* + \bar{x} + b_1 v_{13} + \bar{\theta} \quad (14)
\end{align*}
\]

using the definitions of \( \theta, \Omega^T \), and \( M_{201} = (\theta, \theta) \).

Note that \( \Omega_{\theta}^T, \xi^*, v_{13}, S_{13} \) and \( \bar{x} \) denote \( k \)th row of \( \Omega^T \), \( \xi^* \), \( v_{13} \), \( S_{13} \) respectively and \( e_i \) denotes a vector of an appropriate dimension whose \( k \)th element is one and the remaining elements are zero. We are interested in the trajectory control of \( y=q_1 \). Consider the tracking error \( z_i \) defined as

\[
z_i = q_i - q_{1r} \quad (15)
\]

Now the controller design is performed in two steps following a backstepping design technique of [8]. The derivative of \( z_i \) is

\[
\dot{z}_i = \xi^* + \bar{x} + b_1 v_{13} + \bar{\theta} \quad (16)
\]

Since \( v_{13} \) is treated in (16) as a virtual control for controlling \( z_i \), define \( z_2 \) as

\[
z_2 = v_{13} - \hat{\rho} \hat{q}_i - \alpha_i \quad (17)
\]

where \( \hat{\rho} \) is an estimate of \( \rho-1/\beta \) and \( \alpha_i = \hat{\rho} \bar{\alpha}_i \) is a stabilization function chosen as

\[
\alpha_i = \hat{\rho} \bar{\alpha}_i = \hat{\rho}(c_i z_i - d_i z_i + \xi^* + \bar{\theta}) \quad (18)
\]

where \( c_i, d_i > 0 \) and \( \bar{\theta} \) is an estimate of \( \theta \). Noting that \( b_1 \hat{\rho} = 1 - b_1 \hat{\rho} \), (16) becomes

\[
\dot{z}_i = b_1 z_2 - c_i z_i - d_i z_i + \bar{x} + \bar{\theta} - b_1 \hat{\rho} (\bar{\alpha}_i + \bar{\theta}) \quad (19)
\]

Note that \( q_i \) tracks \( q_{1r} \) if \( z_i = \bar{\theta} = \hat{\rho} = 0 \). Now consider a Lyapunov function of the form

\[
V_i = \frac{1}{2} z_i^2 + \frac{1}{2\gamma} |b_1 \bar{\rho}|^2 + \frac{1}{d_i} z_i^T P \bar{x} \quad (20)
\]

where \( P \) is a positive definite matrix defined in (9). The derivative of \( V_i \) becomes

\[
\dot{V}_i = z_i \dot{z}_i - \frac{1}{\gamma} |b_1 \bar{\rho}|^2 + \frac{1}{d_i} (z_i^T (P^T + P) \bar{x}) \quad (21)
\]

Using (9), (13), and (18), (20) becomes

\[
\dot{V}_i \leq b_1 z_i z_2 - c_i z_i^2 + z_i \bar{\theta} - \frac{3|b_1|^2}{4d_i} \quad (22)
\]

using the following relation:
\[ d_i(z_i - \frac{x_i}{2d_i})^2 = d_i(z_i^2 - z_i\bar{x}_i + \frac{x_i^2}{4d_i}) \geq 0 \quad (22) \]

Since  is unknown, this can be eliminated from (21) by choosing the following update law
\[ \dot{\rho} = -\text{sgn}(b_i)z_i(\bar{\rho} + \dot{q}_i) \quad (23) \]

Substituting the update law (23), (21) becomes
\[ (21) \]

Using (6), (12) and noting that
\[ \begin{aligned}
\dot{a}_i &= \frac{\partial \alpha_i}{\partial q_j} (\dot{\theta} q_j + \dot{q}_j q_i + \dot{\xi}_i + \Omega_{ij} \gamma \dot{\theta}) \\
\dot{a}_i^T &= \frac{\partial \alpha_i}{\partial q_j} (\dot{\theta} q_j + \dot{q}_j q_i + \dot{\xi}_i + \Omega_{ij} \gamma \dot{\theta}) \\
\end{aligned} \]

The unknown 0-dependent term in (24) will be compensated in the following steps:

Consider the derivative of using (17)
\[ \dot{z}_i = \dot{v}_i - \dot{\rho}\dot{q}_i - \dot{\rho}\dot{q}_i - \alpha_i \quad (25) \]

Here  is a function of  and its derivative becomes
\[ \begin{aligned}
\alpha_i &= \beta_i + \frac{\partial \alpha_i}{\partial q_j} \dot{q}_j + \frac{\partial \alpha_i}{\partial q_j} \dot{q}_j \\
\end{aligned} \quad (26) \]

where
\[ \begin{aligned}
\beta_i &= \frac{\partial \alpha_i}{\partial q_j} \dot{v}_i + \frac{\partial \alpha_i}{\partial \rho} \dot{\rho} + \frac{\partial \alpha_i}{\partial \xi_i} \dot{\xi_i} + \frac{\partial \alpha_i}{\partial S_{ij}} \dot{S}_{ij} + \frac{\partial \alpha_i}{\partial q_j} \dot{q}_j + \frac{\partial \alpha_i}{\partial \gamma} \dot{\gamma} \\
\end{aligned} \]

Using (6), (12) and noting that  and  (26) can be expressed as
\[ \begin{aligned}
\dot{a}_i &= \beta_i + \frac{\partial \alpha_i}{\partial q_j} (\theta q_j + \theta q_j + \xi_i + \Omega_{ij} \gamma \dot{\theta} + \bar{x}_i) \\
&\quad + \frac{\partial \alpha_i}{\partial q_j} (\theta q_j + \theta q_j + \xi_i + \Omega_{ij} \gamma \dot{\theta} + \bar{x}_i) \\
\end{aligned} \quad (27) \]

Using the relation of  (27) becomes
\[ \begin{aligned}
\dot{a}_i &= \beta_i + \frac{\partial \alpha_i}{\partial q_j} [\frac{\partial \alpha_i}{\partial q_j} (\bar{x}_i) + \frac{\partial \alpha_i}{\partial q_j} (\Omega_{ij} \gamma + q e_q^T + q e_q^T)] \dot{\theta} \\
&\quad + \frac{\partial \alpha_i}{\partial q_j} [\Omega_{ij} \gamma + q e_q^T + q e_q^T] \dot{\theta} \\
\end{aligned} \quad (28) \]

where  and  are defined by

\[ \dot{V}_i \leq b_i z_i \dot{z}_i - c_i z_i^2 + z_i \ddot{\rho} - \frac{3}{4d_i} |\dot{\theta}|^2 \quad (24) \]

From (10) and (11) of (25) is related to the control input  by
\[ \dot{v}_i = -L_0 v_i + u \quad (29) \]

Eq. (25) becomes
\[ \dot{z}_i = u + a^T \bar{x} - a^T \bar{x} - a^T \ddot{\theta} \quad (30) \]

where
\[ a^T = -L_0 v_i - \dot{\rho}\dot{q}_i - \dot{\rho}\dot{q}_i - \beta_i \]

Consider a Lyapunov function
\[ V_2 = V_i + \frac{1}{2} z_i^2 + \frac{1}{2} \ddot{x}_i^T P \bar{x} + \frac{1}{2} \ddot{\theta}^T \Gamma^{-1} \ddot{\theta} \quad (31) \]

where  is a positive definite symmetric matrix. Using (9), (24), and noting that  becomes
\[ \dot{V}_i = V_i + z_i \dot{z}_i - \frac{1}{d_i} |\dot{\theta}|^2 - \ddot{\theta}^T \Gamma^{-1} \ddot{\theta} \leq -c_i z_i^2 - \frac{3}{4d_i} |\dot{\theta}|^2 + z_i (u + a^T \bar{x} - a^T \ddot{\theta}) \quad (32) \]

In view of (32) the control input is defined as
\[ u = -a^T - c_i z_i - d_i |\dot{\theta}|^2 \quad (33) \]

and (32) becomes
\[ \dot{V}_z \leq -c_z z_z^2 - \frac{3}{4d_z} \| \vec{x} \|^2 - \frac{1}{d_z} \| \vec{x} \|^2 - c_z z_z^2 - d_z z_z^2 \| a_z \|^2 \]
\[ -a_z^T \vec{x} z_z + \vec{b}^T \left[ \sigma z_z + e_z z_z - a_z z_z - \Gamma^{-1} \dot{\theta} \right] \] (34)

Using the relations of
\[ d_z(z_z | a_z| + \frac{\| \vec{x} \|^2}{2d_z}) \geq 0 \]
\[ -d_z(z_z | a_z| - z_z | a_z| \| \vec{x} \|^2 | \leq \frac{\| \vec{x} \|^2}{4d_z} \leq \frac{\| \vec{x} \|^2}{4d_z} \]

Eq. (34) becomes
\[ \dot{V}_z \leq -c_z z_z^2 - c_z z_z^2 - \frac{3}{4} \left( \frac{1}{d_z} + \frac{1}{d_z} \right) \| \vec{x} \|^2 + \vec{b}^T (\tau - \Gamma^{-1} \dot{\theta}) \] (35)

where
\[ \tau = \sigma z_z + e_z z_z - a_z z_z \]

Now we can choose an adaptation law for \( \dot{\theta} \) as
\[ \dot{\theta} = \Gamma \tau \] (36)

which yields
\[ \dot{V}_z \leq -c_z z_z^2 - c_z z_z^2 - \frac{3}{4} \left( \frac{1}{d_z} + \frac{1}{d_z} \right) \| \vec{x} \|^2 \]

In summary, for the closed-loop system described by (4), (23), (33), and (36), the tracking error \( q_1 - q_{1r} \) tends to be zero if \( q_{1r} \) is a bounded and smooth trajectory.

5 Simulation Results

In this section, numerical results for the link angle control is presented. The simulation parameter are given in Table 1. Fig. 2 shows the block diagram of the control scheme simulated by the Matlab and Simulink. Fig. 3 shows the plot of link angle for a smooth trajectory generated by the parameters shown in Table 1. Fig. 4 shows the plot of motor angle and Fig. 5 shows the plot of control torque.

![Fig. 2 Controller Block Diagram](image)

As shown in this simulation, a command trajectory can be properly chosen to shape the transient responses. After extensive simulation a suitable choice of parameters \( c_z, d_z, L, \gamma, \Gamma \) can be selected for the desired closed-loop responses. For smaller \( c_z \), the control input magnitude is reduced for poor tracking capability. Here only selected simulation results are presented. Fig. 6 and Fig. 7 show results of adaptation laws for \( \rho \) and \( \theta \).
References