Measuring Data Based Non-linear Error Modeling for Parallel Machine Tool

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Abstract
Converting a nonlinear problem to a linear one by means of the least square fit, a nonlinear error modeling method based on measuring data is presented. Combined with an example, some key items are pointed out during modeling. The simulation results on parallel machine tool show that the model based on the method is of high accuracy and the error modeling method is correct and reliable. No matter what error parameter of position and orientation is, the ideal error model would be obtained by means of the method. The accuracy of parallel machine tool can be raised greatly by using the model to compensate the position and orientation.

Key words: Nonlinear error, modeling, parallel machine tool, accuracy compensation

Introduction
The research on parallel machine tool has closely attracted scholar’s attention in recent years[1-10]. Therefore, some problems should be solved before practical application[11-12]. Of all these problems, precision must be faced up to. Because of no effective method to measure the position and orientation of parallel machine tool on-line directly, the full closed loop control cannot be realized. Usually, precision design and kinematics calibration are used to improve the precision of parallel machine tool. Because manufacturing error is not the only reason that affects the precision of the machine tool, it is difficult to achieve great development on the precision of parallel machine tool by these means.

Error compensation is an effective means to raise the precision of parallel machine tool[12]. The focus is to find error characteristics and seek a method to compensate the error under opening control mode system. The core technology is error modeling and the parameters identification of the model based on measuring data. By means of the model and compensation algorithm, it can receive the information from sensors, estimate processing conditions, decide proper action and transfer order to the actuators. Hence, the processing precision of the machine tool can be limited within the tolerance. Because many factors can cause errors, in fact, it is very difficult to quantify every factor that affects the error of the position and orientation of parallel machine tool movable platform. Real compensation is to correct the machine tool position and orientation according to its current condition in real time. From this point of view, a nonlinear error modeling method based on measuring data is presented. The error function between measuring information and theoretical model is set up. A nonlinear problem is converted to a linear one and the model parameters are identified by means of the least square fit method. Finally, the error model of the parallel machine tool is set up. Simulation results show that the model is of high accuracy and the error modeling method is correct and reliable.

1 Error modeling method and parameters identification
Knowing that the errors of parallel machine tool in all directions change with variation of the position and
orientation of movable platform, assume that the error of the 
machine tool is the function of the position and orientation. 
The relation between them is nonlinear according to the 
measuring data analysis. Take the error in \( x \) direction for 
example to illustrate the process of modeling. Construct 
polynomial error model of the position and orientation as 
follows:

\[
f(x, y, z, A, B, C) = a_0 + a_1 x + a_2 y + a_3 z + a_4 x^2 + a_5 y^2 + a_6 z^2 + a_7 x y + a_8 x z + a_9 y z + a_{10} A + a_{11} B + a_{12} C + a_{13} x A + a_{14} x B + a_{15} x C + a_{16} y A + a_{17} y B + a_{18} y C + a_{19} z A + a_{20} z B + a_{21} z C + a_{22} y A + a_{23} y B + a_{24} y C + a_{25} A^2 + a_{26} B^2 + a_{27} C^2 + a_{28} A B + a_{29} A C + a_{30} B C + \ldots
\]

(1)

In formula (1), let:

\[
\begin{align*}
 x_0 &= 1, \quad x_1 = x, \quad x_2 = y, \quad x_3 = z, \quad x_4 = A, \quad x_5 = B, \quad x_6 = C, \\
 x_7 &= x^2, \quad x_8 = y^2, \quad x_9 = z^2, \quad x_{10} = A^2, \quad x_{11} = B^2, \quad x_{12} = C^2, \quad x_{13} = x y, \quad x_{14} = x z, \\
 x_{15} &= x A, \quad x_{16} = x B, \quad x_{17} = x C, \quad x_{18} = y z, \quad x_{19} = y A, \quad x_{20} = y B, \quad x_{21} = y C, \\
 x_{22} &= z A, \quad x_{23} = z B, \quad x_{24} = z C, \quad x_{25} = A B, \quad x_{26} = A C, \quad x_{27} = B C, \quad \ldots
\end{align*}
\]

Thus formula (1) can be written in form of multielement 
simple equations:

\[
\Delta x = \sum_{i=0}^{n} a_i x_i
\]

(2)

In formula (2), \( n \) is the number of the polynomial items. The 
model parameters can be identified by means of the least 
square fit method. Substitute the measuring data into the 
formula and calculate square sum of residues:

\[
S = \sum_{i=1}^{m} \left[ \Delta x_i - \sum_{i=0}^{n} a_i x_i \right]^2
\]

(3)

\( m \) is the number of measuring data. Make partial derivative 
in turn. Let:

\[
\frac{\partial S}{\partial a_j} = -2 \sum_{i=1}^{m} \left[ \Delta x_i - \sum_{i=0}^{n} a_i x_i \right] x_j = 0
\]

(4)

\( (i=0,1,2,3,\ldots,m; \quad k=1,2,3,\ldots,n) \)

Formula (4) can be written in form as follows:

\[
G P^T = Q
\]

(5)

Here: \( G \)-- \((n+1)\times(n+1)\) rank coefficient matrix

\[
G(i, j) = -2 \sum_{k=1}^{m} x_k x_j
\]

\[
P=(a_0, a_1, a_2, a_3, \ldots, a_n)
\]

\[
Q(i) = -2 \sum_{i=1}^{m} \Delta x_i x_i
\]

\[ (j=0,1,2,3,\ldots,m; \quad i=0,1,2,3,\ldots,n) \]

The error model is a nonlinear system to the position and 
orientation of the movable platform. However, formula (5) is 
just a linear system to \( P \). In this way, a complicated nonlinear 
problem has been converted to a simple linear one. Solve 
formula (5), and then get \( P \)-- identification value of model 
parameters. Substitute \( P \) to formula (1). That is the error 
model of the movable platform position and orientation. 
The flow chart of identification of error model is shown in 
Fig. 1.

2 Example

An example is given as follows to verify the accuracy of 
model set up by the above method:

\[
y = \frac{86 + 2 x_1 - 3 x_2 + 6 x_3 - 7 x_1^2 - 5 x_2^2 + 10 x_3^2}{25 x_1 x_2 + 13 x_1 x_3 - 9 x_2 x_3 + 2 x_1^2}
\]

Construct a polynomial error model

\[
y = f(x_1, x_2, x_3)
\]

According to the value \( y \) of 125 points which is symmetrical
to origin \((x_1=10, -5, 0, 5, 10, x_2=-5, -2.5, 0, 2.5, 5, x_3=-7, -3.5, 0, 3.5, 7)\), the model parameters are identified. Substitute value \(y\), and then output coefficient vector \(P\) (keep 8 digits after decimal point).

\[
\]

Comparing the coefficient of original polynomial with model polynomial, the difference of every corresponding term is very small, so the model is of high accuracy. Substitute \(P\) into the model and calculate simulation results. The difference between simulation and real value reflects the model accuracy more directly.

\[
\Delta = y - y_0
\]

\[
\delta = \frac{\Delta}{y_0}
\]

Here: \(\Delta\) --The difference between simulation and real value; 
\(\delta\) --relative difference between simulation and real value; 
\(y\)--simulation result; 
\(y_0\)--calculation result of original polynomial (real value).

A great deal of calculation shows that the accuracy of sample point in the boundary limited by original parameters is \(\Delta \leq 1.351310^6\); the accuracy of non-sample point in the boundary is \(\Delta \leq 4.53310^6\); the accuracy of out-boundary point (within 3 times size of original boundary) is \(\Delta \leq 5.83310^5\). The example verified that the accuracy of the model constructed by method mentioned above is of high accuracy. Above all, some key items should be noticed carefully as follows:

1. The boundary limited by original parameters should be symmetric boundary about coordinate origin. If it is a non-symmetric boundary, convert it into a symmetric one by coordinate transformation. Otherwise, the constant term will change as the original boundary varies. Furthermore, it causes the other term coefficients to change, thus the model accuracy decreases.

2. There must be 3 different values of all parameters in original data at least (usually, take 5). The longer the data serial is, the higher the model accuracy.

3. For the reason of the internal accuracy being much higher than the external one, the boundary limited by origin parameters should be close to the actual boundary as much as possible.

### 3 Error modeling of six-bar linkage in the parallel machine tool

#### 3.1 Error of six-bar linkage

The five-axis parallel machine tool is shown in Fig. 2. Error of the six-bar linkage, of which the top edge is connected with the movable platform by joints, is the deviation of the actual movement plane to the ideal movement plane of the middle point on the top edge of the six-bar linkage. The results of theoretical research and experimental analysis show that the deviation changes with the variation of the position and orientation parameters \(y, z\) and \(A\). Repeat measuring the error of the six-bar linkage and average them, as shown in Tab. 1.
Fig. 3 (a), (b), (c) respectively show the relation between the error of six-bar linkage of the parallel machine tool in coordinate $x$ direction and one of the 3 parameters ($y$, $z$, $A$) when the other two are constant. All of the relations are nonlinear.

### 3.2 Error modeling of six-bar linkage and results of simulation

Construct the 2-order, 3-order, 4-order error model of six-bar linkage respectively:

\[
\Delta x_2 = f_2(y, z, A) \\
\Delta x_3 = f_3(y, z, A) \\
\Delta x_4 = f_4(y, z, A)
\]

According to the above method, translate coordinate axis $z$ from $z=0$ to $z=350$ to make the boundary symmetrical about the origin. Correspondingly, substitute $z=350$ for $z$ in the model obtained. The result of error model parameters identification is shown in Tab. 2. Calculate the difference $\delta_i$ between simulation results and measuring data

\[
\delta_i = \Delta x - \Delta x_i
\]

here; $\Delta x_i$: simulation result

$\Delta x_i$: measuring data

The maximum position error of parallel machine tool movable platform caused by the six-bar linkage is 0.520mm in Tab. 1 while Tab. 3 shows that the model simulation errors of each order are: $\Delta x_2 \leq 0.011$mm, $\Delta x_3 \leq 0.006$mm, $\Delta x_4 \leq 0.004$mm respectively. Tab. 3 indicates that the accuracy of simulation results changes as the model order varies. The higher the model order is, the more accurate its simulation result. That is to say, the accurate model can be obtained by adjusting model orders according to actual demand. Thus it can be proven that the error modeling method above is correct and reliable. Comparing Tab. 1 with Tab. 3, it is obvious that using the model to compensate the position an

### Table 1

<table>
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<tr>
<th>x</th>
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<th>T</th>
<th>Y</th>
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\[
\Delta x_1 = \Delta x_2 = \Delta x_3 = \Delta x_4
\]
orientation of parallel machine tool movable platform in \( \chi \) direction can raise the accuracy of it greatly. Having universality, the method can be used to get accurate error model in any similar case. No matter what error parameters of position and orientation is, the ideal error model will be obtained by means of the method above. Using the error model to compensate the corresponding position and orientation of parallel machine tool movable platform in real time can raise the orientation accuracy of machine tool in related coordinate direction.

### Parameters

**Tab. 2** Model parameters identification results

<table>
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<tr>
<th>Parameters</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
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<td>1.817\times10^{-3}</td>
<td>1.706\times10^{-6}</td>
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**Tab. 3** The difference between simulation results and measuring data (unit: z-mm, y-mm, \( \delta \)-\( \mu \)-m)

<table>
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<tr>
<th>( z )</th>
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4 Conclusion

(1) A nonlinear error modeling method based on measuring data is presented. The model parameters are identified by means of the least square fit method, thus a complicated nonlinear problem is converted to a simple linear one.

(2) Combining with an example, it pointed out that the boundary limited by original parameters should have such characteristics as: ① It is symmetric boundary about coordinate origin. If it is a non-symmetric one, convert it into symmetric one by coordinate transformation. Otherwise, it will decrease the model accuracy. ② In original data there must be at least 3 different values of every corresponding parameter. ③ For the internal accuracy is much higher than the external one, it should be close to the actual boundary as much as possible.

(3) The simulation result of error model of six-bar linkage of parallel machine tool with 5 DOF shows that the error model based on the above method is accurate. The modeling method is correct and reliable.

(4) Using the model to compensate the position and orientation of parallel machine tool movable platform can raise the accuracy of it greatly.

Acknowledgement

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Abstract
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Key words: Nonlinear error, modeling, parallel machine tool, accuracy compensation

Summary
A nonlinear error modeling method based on measuring data is presented. The model parameters are identified by means of the least square fit method, thus a complicated nonlinear problem is converted to a simple linear one. (2) Combining with an example, it pointed out that the boundary limited by original parameters should have such characteristics as: (1) It is symmetric boundary about coordinate origin. If it is a non-symmetric one, convert it into symmetric one by coordinate transformation. Otherwise, it will decrease the model accuracy. (2) In original data there must be at least 3 different values of every corresponding parameter. (3) For the internal accuracy is much higher than the external one, it should be close to the actual boundary as much as possible. The simulation result of error model of six-bar linkage of parallel machine tool with 5 DOF shows that the error model based on the above method is accurate. The modeling method is correct and reliable. Using the model to compensate the position and orientation of parallel machine tool movable platform can raise the accuracy of it greatly.

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