Model-free Regulation of Multi-link Smart Materials Robots

S. S. Ge, T. H. Lee and Z. P. Wang
Department of Electrical and Computer Engineering
National University of Singapore
Singapore 117576

Abstract: In this paper, model-free controllers are presented for multi-link smart materials robots. The controllers are derived from the basic energy-work relationship in the absence of the system model which is complex and difficult to obtain for multi-link smart materials robots. The smart materials bonded along the links are used to apply additional control to suppress the residue vibration effectively. One can achieve not only the closed-loop stability of the original system, but also the asymptotic stability of the truncated system, which is obtained through representing the deflection of each link by an arbitrary finite number of flexible modes. Simulation results are provided to show the effectiveness of the presented approach.

1 Introduction

For high speed positioning applications, light-weight manipulators are of considerable interest. Much attention has been given to modeling and control of flexible-link manipulators. The majority of the research effort has been on position control of the end-effector through actuation from the joint motors. Based on a truncated model obtained from either Finite Element Method (FEM) or Assumed Modes Method (AMM), various kinds of control approaches have been applied to improve the performance of flexible systems [1]-[5]. To avoid the problems associated with the truncated model, such as controller/observer spillover problems [6], some controllers are designed based on the Partial Difference Equations (PDEs) directly [6] [7]. While most research is dealing with single flexible link case for its simplicity, some work for multi-link flexible link robots have also been studied [5] [8].

Recently, utilization of smart or intelligent materials in the control of flexible structures is receiving increased attention. Since piezoelectric materials can be bonded/embedded along a beam and achieve the transformation between mechanical deformation and electrical field, they are ready to serve as actuators and sensors [9]-[11]. In this paper, by combining the results in [8] for multi-link flexible robots and that in [11] for single link smart materials robot, we present a class of model-free controller for multi-link smart materials robots. The controllers are developed based on the basic energy-work relationship, subsequently avoid the problems of model-based methods. The controllers can guarantee the stability of the closed-loop system and are very robust in terms of system parameter variations.

2 System Description

We shall consider a N-link smart materials robot (i) deployed in space, or (ii) moving in a horizontal plane. In both cases, the effect of gravity is ignored for simplicity.

As shown in Figure 1, the N links are connected by N motors. Motor 1 is fixed at the origin of the inertial frame and the rest motors serially connect the N links. Let \( \theta_i \) denotes the desired joint angular position for link \( i \), while \( \theta_i \) is the actual joint angle, \( \tau_i \) is the torque applied to the joint. Assuming \( n_i \) pairs of piezoelectric sensors and actuators collocated at \( n_i \) points along link \( i \), \( \mathbf{v}_i = [v_{i1} \, v_{i2} \, \ldots \, v_{in_i}]^T \in \mathbb{R}^{n_i} \) is the vector of control voltages applied to actuators, while \( \mathbf{I}_i = [I_{i1} \, I_{i2} \, \ldots \, I_{in_i}]^T \in \mathbb{R}^{n_i} \) is the vector of the electrical currents induced by sensors, \( I_{ij} \) is proportional to the change of strain [12].

Hence, the total work done by external inputs \( W = \sum_{i=1}^{N} \int_0^T \tau_i(t) \dot{\theta}_i(t) dt + \sum_{i=1}^{N} \int_0^T \mathbf{v}_i^T \mathbf{I}_i dt \), which consists of that done by the torque applied to the N joints and that contributed by the voltages applied to piezoelectric actuators. Damping and friction are neglected because their exclusion will not affect the closed-loop stability. Thus, from the energy-work relationship, we have the following equation

\[
E_k(t) + E_p(t) - [E_k(0) + E_p(0)] = \sum_{i=1}^{N} \int_0^T \dot{\theta}_i(t) \tau_i(t) dt + \sum_{i=1}^{N} \int_0^T \mathbf{I}_i^T \mathbf{v}_idt
\]

(1)

where \( E_k(t) \) and \( E_k(0) \), \( E_p(t) \) and \( E_p(0) \) are the total kinetic energy and total potential energy of system at time \( t \) and 0, respectively. Then we have

\[
\dot{E}_k(t) + \dot{E}_p(t) = \sum_{i=1}^{N} \dot{\theta}_i(t) \tau_i(t) + \sum_{i=1}^{N} \mathbf{I}_i^T \mathbf{v}_i
\]

(2)
3 Model-Free Controller Design

In this section, two model-free controllers are presented for multi-link smart materials robots. The control objective here is to rotate each link of the robot to the desired angular position and simultaneously suppress the residual vibrations effectively.

3.1 Decentralized model-free controller

In certain case, decentralized controllers are very desirable because of (i) the technical difficulties in implementing communications among the controllers located at different places; and (ii) the inherent robustness of a decentralized control system.

Consider the following decentralized model-free controller (DMFC)

\[
\mathbf{u}_i = \begin{bmatrix} \tau_i \\ \mathbf{v}_i \end{bmatrix} = \begin{bmatrix} -k_{pi}[\theta_i(t) - \theta_{di}] - k_{vi}\dot{\theta}_i(t) \\ -\mathbf{K}_{psi} \int_0^t \mathbf{I}_d dt - \mathbf{K}_{usi} \mathbf{I}_s_i \end{bmatrix}
\]  

(3)

where \( \mathbf{K}_{psi} = \text{diag}[k_{psi,i}] \in \mathbb{R}^{n_i \times n_i} \), \( \mathbf{K}_{usi} = \text{diag}[k_{usi,i}] \in \mathbb{R}^{n_i \times n_i} \), and \( k_{pi}, k_{vi}, k_{usi} > 0 \), \( k_{psi,i} \geq 0 \), \( i = 1, 2, \ldots, N \).

The controller is clearly decentralized because the control signals \( \tau_i \) and \( \mathbf{v}_i \) for the \( i \)-th link depends on their respective local feedback information pairs \( (\theta_i(t), \dot{\theta}_i(t)) \) and \( \left( \int_0^t \mathbf{I}_d dt, \mathbf{I}_s_i \right) \).

Theorem 3.1. DMFC (3) can stabilize the multi-link smart materials robot system.

Proof. Consider the following Lyapunov function candidate

\[
V_1(t) = E_k(t) + E_p(t) + \frac{1}{2} \sum_{i=1}^N k_{pi}[\theta_i(t) - \theta_{di}]^2 + \frac{1}{2} \sum_{i=1}^N \left( \int_0^t \mathbf{I}_d dt \right)^T \mathbf{K}_{psi} \left( \int_0^t \mathbf{I}_d dt \right)
\]  

(4)

It is a Lyapunov function candidate because the energy of system \( E_k(t) + E_p(t) \geq 0 \).

By virtue of equations (2) and (3), the time derivative of \( V_1 \) is given by

\[
\dot{V}_1(t) = \sum_{i=1}^N \dot{\theta}_i(t) \tau_i(t) + \sum_{i=1}^N \mathbf{I}_d^T \mathbf{v}_i + \sum_{i=1}^N k_{psi,i} \left[ \int_0^t \mathbf{I}_d dt \right] \frac{\partial}{\partial \theta_i} \left[ \frac{1}{2} k_{psi,i} \theta_i^2 \right] + \sum_{i=1}^N k_{vi} \dot{\theta}_i^2 + \sum_{i=1}^N k_{psi,i} \left( \int_0^t \mathbf{I}_d dt \right)^2 + \sum_{i=1}^N k_{psi,i} \left( \int_0^t \mathbf{I}_d dt \right)^2
\]

\[
= -\sum_{i=1}^N k_{psi,i} \dot{\theta}_i^2 - \sum_{i=1}^N \mathbf{I}_d^T \mathbf{K}_{psi} \left( \int_0^t \mathbf{I}_d dt \right)
\]

\[
\leq 0
\]

(5)

therefore, the closed-loop system is stable in the sense of Lyapunov.

3.2 Centralized model-free controller

In order to further improve the control performance and consider those situations that centralized controllers can be implemented, we shall study the centralized model-free controller (CMFC) design.

Consider the following CMFC

\[
\mathbf{u}_i = \begin{bmatrix} \tau_i \\ \mathbf{v}_i \end{bmatrix} = \begin{bmatrix} -k_{psi}[\theta_i(t) - \theta_{di}] - k_{vi}\dot{\theta}_i(t) - \tau_{ci} \\ -\mathbf{K}_{psi} \int_0^t \mathbf{I}_d dt - \mathbf{K}_{usi} \mathbf{I}_s_i - \mathbf{v}_{ci} \end{bmatrix}
\]  

(6)

where \( \tau_{ci} = \sum_{j=1}^{m_i} k_{bi,j} f_{bi,j}(t) \int_0^t \dot{\theta}_j(s) f_{bi,j}(s) ds \) and \( \mathbf{v}_{ci} \) is a column vector defined as

\[
\mathbf{v}_{ci} = \begin{bmatrix} \sum_{j=1}^{m_i} k_{bi,j} f_{bi,j}(t) \int_0^t \dot{\theta}_j(s) f_{bi,j}(s) ds \\ \vdots \\ \sum_{j=1}^{m_i} k_{bi,j} f_{bi,j}(t) \int_0^t \dot{\theta}_j(s) f_{bi,j}(s) ds \end{bmatrix} \in \mathbb{R}^{n_i}
\]

\( f_{bi,j}, f_{ik,j} \) are any time integrable signals, \( \mathbf{K}_{psi} = \text{diag}[k_{psi,i}] \in \mathbb{R}^{n_i \times n_i}, \mathbf{K}_{usi} = \text{diag}[k_{usi,i}] \in \mathbb{R}^{n_i \times n_i}, k_{psi}, k_{vi}, k_{usi} > 0, k_{psi,i} \geq 0, i = 1, 2, \ldots, N. \)

In comparison with (3), it is the presence of \( \tau_{ci} \) and \( \mathbf{v}_{ci} \) makes (6) a centralized controller for \( f_{bi,j} \) and \( f_{ik,j} \) can be signals from others locations of the corresponding link or from other links.

Theorem 3.2. CMFC (6) can stabilize the multi-link smart materials robot system.

Proof. Consider the following Lyapunov function candidate

\[
V(t) = V_1(t) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^{m_i} k_{bi,j} \left( \int_0^t \dot{\theta}_j(s) f_{bi,j}(s) ds \right)^2 + \frac{1}{2} \sum_{i=1}^N \sum_{k=1}^N \sum_{j=1}^{m_{ik}} k_{ik,j} \left( \int_0^t I_{ik}(s) f_{ik,j}(s) ds \right)^2
\]  

(7)

where \( V_1(t) \) is defined in equation (4).

Taking the time derivative of \( V \) and applying equation (2), we arrive at

\[
\dot{V}(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^{m_i} k_{bi,j} \left( \int_0^t \dot{\theta}_j(s) f_{bi,j}(s) ds \right)^2 + \frac{1}{2} \sum_{i=1}^N \sum_{k=1}^N \sum_{j=1}^{m_{ik}} k_{ik,j} \left( \int_0^t I_{ik}(s) f_{ik,j}(s) ds \right)^2
\]

(8)

Therefore, the closed-loop system is stable in the sense of Lyapunov.
\[ \dot{V}(t) = \sum_{i=1}^{N} \tau_i(t) \dot{\theta}_i(t) + \sum_{i=1}^{N} \sum_{k=1}^{n_i} v_{ik}(t) I_{ik}(t) + \sum_{i=1}^{N} k_{pi} \]

\[ [\theta_i(t) - \theta_d] \dot{\theta}_i(t) + \sum_{i=1}^{N} \sum_{k=1}^{n_i} k_{pi} \int_0^t I_{ik}(s) ds I_{ik}(t) \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{n_i} k_{ki} \dot{\theta}_i(t) f_{ij}(t) \int_0^t \dot{\theta}_i(s) f_{ij}(s) ds \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{n_i} k_{kj} I_{ik}(t) f_{jk}(t) \int_0^t I_{ik}(s) f_{jk}(s) ds \]  

(8)

Substituting equation (6) into equation (8) yields

\[ \dot{V}(t) = - \sum_{i=1}^{N} k_{ii} \dot{\theta}_i(t)^2 - \sum_{i=1}^{N} \sum_{k=1}^{n_i} k_{v,ik} I_{ik}^2(t) \leq 0 \]

which means that the closed-loop system is stable in the Lyapunov sense.

Remarks:

1. For DMFC (3), with the aid of local voltage control, the vibration of the flexible link can be controlled directly and the control performance can be improved. In an attempt to include explicit evaluation of the vibration into the controller, nonlinear terms \( \tau_{cl} \) and \( v_{ci} \) are introduced into CMFC (6). Theoretically, the stability of the system will not be affected for any time integrable functions \( f_{ii}, f_{kj} \), but it is preferable to select them to be associated with the vibration of the flexible link. The joint PD controller for link \( i \) is a special case of the \( \tau_i \) in (6) by setting \( k_{ij} \)'s to be zeros.

2. All the controllers (3) and (6) are independent of system parameters and thus possesses stability robustness to system parameters uncertainties. The stability proof is independent of the system dynamics and thus the problem associated with model-based controllers mentioned in Section 1 Introduction are avoided.

3. All the controllers are very easy to implement in practice. \( f_{ikj} \) and \( f_{ii} \) can be chosen as any variable or any combination of variables related to the vibration of the flexible link. For example, in the torque control of link 1, \( f_{i1}(t) \) can be signals from the second link.

Although the stability of the closed-loop system has been given, to prove the asymptotic stability of the system is not easy due to the infinite dimensionality of the system. If the system is of finite dimensions, then LaSalle’s theorem can be used to prove the asymptotic stability.

**Theorem 3.3.** Controller (3) can guarantee the asymptotic stability of the damped truncated system where the flexible deflection is described by arbitrarily any finite number of flexible modes. Furthermore, controller (6) can also guarantee the asymptotic stability of the same truncated system if \( f_{kj}, f_{ij} \) are selected as the functions that are equal zero when the smart materials robot is undeformed.

**Proof:** Using the same Lyapunov functions (4) and (7), we consider the motion of the system in the largest invariant set in the set \( V_i = 0 \) \( (\dot{V} = 0) \). In both cases, we have \( \dot{\theta}_i \equiv 0 \) and \( I_{ii} \equiv 0 \). Hence, there is no energy input to the system since \( \dot{\theta}_i \tau_i + I_{ii}^2 \dot{\omega}_i = 0 \). Subsequently, \( \dot{\theta}_i = 0, \tau_i = -k_{pi}(\dot{\theta}_i - \theta_{di}), v_i = 0, \ i = 1, 2, \ldots, N \).

Firstly, consider link 1. With the aid of Hamilton’s principle, considering the motion of system in \( \dot{V}_1 = 0 \) \( (\dot{V} = 0) \), we have the following PDEs of link 1

\[ c_{c1} w''_1(t, 0) - k_{pi} (\dot{\theta}_1 - \theta_{d1}) = 0 \]

\[ \rho_{L1} \ddot{w}_1(x, t) = -c_{c1} w'''_1(x, t), 0 \leq x_1 \leq L_1 \]

and boundary conditions (BCs) of link 1

\[ w_1(0, t) = 0, \quad w_1'(0, t) = 0 \]

\[ w_1''(0, t) = \frac{k_{pi}}{c_{c1}} (\dot{\theta}_1 - \theta_{d1}) \]

Because \( \dot{\theta}_i \equiv 0 \), the robot is operated as if all the motors are locked. Consequently, each motor can be taken simply as a concentrated mass. Hence, the bending moment at the tip of link \( i - 1 \) should be equal to the base bending moment of link \( i \), i.e.

\[ c_{c,i-1} w''_{i-1}(L_{i-1}, t) = c_{c1} w''_1(0, t), \quad i = 2, 3, \ldots, N \]

Considering links 1 and 2, and noting that

\[ c_{c2} w''_2(0, t) = -c_2 = k_{p2} (\dot{\theta}_2 - \theta_{d2}) \]

thus, we have another boundary condition

\[ w_2''(L_1, t) = \frac{k_{p2}}{c_{c1}} (\dot{\theta}_2 - \theta_{d2}) \]

It is noted that the left-hand sides of equations (12) and (13) are functions of time, while the right-hand sides are constants. Let us firstly assumed that the two constants are zero. It is shown later that either constants being non-zero leads to invalid solutions. Now equations (12) and (13) can be rewritten as

\[ w''_1(0, t) = 0, \quad w''_1(L_1, t) = 0 \]

Using the Method of Separating Variables, the solution to (10) is assumed to be of the form \( w(x, t) = \Psi(x) Q(t) \), and equation (10) becomes

\[ \Psi'''(x) \frac{c_{c1}}{\rho_{L1}} = -\ddot{Q}(t) \frac{Q(t)}{Q(t)} \]

where primes denote the derivatives of \( x \) and dots denote the derivatives of \( t \). It is clear that the left hand side of equation (15) is a pure function of \( x \) while the right hand side depends on \( t \) only. Therefore,

3873
both sides of equation (15) should be equal to a constant. If $k$ is used to denote the constant, (15) can be reduced into two ordinary differential equations (ODEs), namely

\[
\ddot{Q}(t) = -kQ(t) \tag{16}
\]

\[
\Psi''(x_1) = \frac{\rho L^1}{c_{e1}} k\Psi(x_1) \tag{17}
\]

The BCs become

\[
\Psi(0) = 0, \: \Psi'(0) = 0, \: \Psi''(0) = 0, \: \Psi''(L_1) = 0 \tag{18}
\]

We will consider equation (17) and conditions (18) with regard to different values of the constant $k$.

It can be proved that when $k = 0$ and $k > 0$, the solution to equation (17) is trivial. Then we just need to consider the case for $k < 0$.

Letting $k = -\omega^2 < 0$, equation (17) can be rewritten as

\[
\Psi''(x_1) = -\left(\frac{\nu}{L}\right)^4 \psi(x_1) \tag{19}
\]

with \( \left(\frac{\nu}{L}\right)^4 = \omega^2 e_{c1}^4 \).

The general solution to equation (19) is of the form

\[
\Psi(x_1) = C_1 e^{ax_1} \sin(ax_1) + C_2 e^{ax_1} \cos(ax_1) + C_3 e^{-ax_1} \sin(ax_1) + C_4 e^{-ax_1} \cos(ax_1) \tag{20}
\]

where \( a = \nu/\sqrt{2L_1} \).

From BCs (18), a set of equations are obtained

\[
\begin{align*}
C_2 + C_4 &= 0 \\
C_1 + C_2 + C_3 - C_4 &= 0 \\
C_1 - C_3 &= 0 \\
(C_1 e^{aL_1} - C_2 e^{-aL_1}) \cos(aL_1) - (C_3 e^{aL_1} - C_4 e^{-aL_1}) \sin(aL_1) &= 0
\end{align*} \tag{21}
\]

To obtain nontrivial solutions, the determinant of the coefficient matrix of equations (21) must be zero, i.e.,

\[
4(\sin(aL_1) \cosh(aL_1) + \cos(aL_1) \sinh(aL_1)) = 0
\]

which may be satisfied by an infinite number of $a$. Consider only positive $a_i$, an infinite number of solutions to the boundary value problem can be given by

\[
\Psi_i(x_1) = C_i e^{a_i x_1} \sin(a_i x_1) + C_2 e^{a_i x_1} \cos(a_i x_1) + C_3 e^{-a_i x_1} \sin(a_i x_1) + C_4 e^{-a_i x_1} \cos(a_i x_1)
\]

where $C_i \sim C_i$ denote the solution to equation (21) corresponding to $a_i$.

When $k = -\omega^2$ with $\omega$ being non-zero number, the solution to equation (16) can be obtained

\[
q_i(t) = D_i^1 e^{a_i t} + D_i^2 e^{-a_i t} \tag{22}
\]

where $D_i^1$ and $D_i^2$ are related to the initial conditions of $y_i(x_1, t)$. Note that the “initial” moment $t_0$ should denote the moment when the system motion enters the invariant set, rather than the initial operating moment since we are considering the motion of the system in the largest invariant set in the set $V_1 = 0 \quad (V = 0)$. Then from the Superposition or Linearity Principle, a solution $w(x, t)$ can be given by

\[
w(x_1, t) = \sum_{i=1}^{\infty} q_i(x_1) q_i(t) \tag{23}
\]

Note that the $\omega_i$ in equation (22) can be either positive or negative, without loss of generality, let $\omega_i > 0$. This leads to $D_1^1 = 0$ as follows. If $D_1^1 \neq 0$, then $\lim_{t \to \infty} q_i(t) \to \infty$ and $\lim_{t \to \infty} \dot{q}_i(t) \to \infty$, and hence $\lim_{t \to \infty} \ddot{w}_1(x_1, t) \to \infty$. This implies the kinetic energy $E_k$ of the system approaches infinity, which contradicts the fact the $V_1$ in equation (4) ($V$ in equation (7)) is actually bounded. Therefore, $D_1^1$ must be zero. Consequently, when $k < 0$, the solution (23) approaches zero as time approaches infinity.

In summary, $w_1(x_1, t) = 0$ provided that the system motion is in the largest invariant set $V_1 = 0 \quad (V = 0)$. Moreover, recalling that we already have $\theta_1 = \theta_{d1}$, we further conclude that if the system motion is in the largest invariant set $V_1 = 0 \quad (V = 0)$, the first link must stop in the final position described by $\theta_1 = \theta_{d1}$ and $w_1(x_1, t) = 0$. Thus, the local frame $O_1X_1Y_1$ is actually static with respect to the inertia frame $O_1X_1Y_1$. This allows us to take $O_2X_2Y_2$ as the inertia frame in which the second link is to be considered. Then we have the following PDEs of link 2

\[
c_{c2} w_2''(0, t) - k_{c2} (\theta_2 - \theta_{d2}) = 0 \tag{24}
\]

\[
\rho_{L2} \ddot{w}_2(x_2, t) = -c_{c2} w_2''(x_2, t), \quad 0 < x_2 < L_2 \tag{25}
\]

Moreover, from Figure 1 and the above conclusions, a set of boundary conditions similar to (11) and (14) also exist for equation (25), i.e.

\[
w_2(0, t) = 0, \: w_2'(0, t) = 0, \: w_2''(0, t) = 0, \: w_2''(L_2, t) = 0
\]

Thus carrying out similar analysis for link 2 leads to the conclusion that link 2 must stop at its final position described by $\theta_2 = \theta_{d2}$ and $w_2(x_2, t) = 0$ provided that the system motion is in the largest invariant set $V_1 = 0 \quad (V = 0)$. This implies the local frame $O_2X_2Y_2$ can be considered as the inertia frame for link 3. Now it is easy to see that repeating the same procedure until link $N$ finally leads to the fact that if the system motion is in the largest invariant set $V_1 = 0 \quad (V = 0)$, then all links must stop at their final positions, i.e. $\theta_i = \theta_{di}$ and $w_i(x_i, t) = 0 \quad (i = 1, 2, \ldots, N)$.

Now, let us consider the case when the right-hand side of either equation (12) or (13) is a non-zero constant is discussed. If this is true, then from $w_1(x_1, t) = \Psi(x_1) Q(t), \: Q(t)$ and hence $w_1(x_1, t)$ must be constant. This means that the first link is static,
which implies that the right side of both equations (12) and (13) must equal the same non-zero constant (from the moment balance of a static bending beam). Subsequently, frame $O_2X_2Y_2$ can be taken as the inertial frame for link 2. Because the base bending moment of link 2 equals the tip bending moment of link 1 (note that the condition $V_1 = 0$ or $V = 0$), the base bending moment of link 2 must be the same constant. This, by assuming $w_2(x_2,t) = \Psi(x_2)\Phi(t)$, implies link 2 is also static. Repeating the same procedures for all links leads to a static bending $N$-link robot. Then from the moment balance of the static robot, the base bending moment of link 1 should be equal to the tip bending moment of link $N$. Since the free tip of link $N$ is loaded with a concentrated mass, the tip bending moment is zero. Therefore either the base bending moment of link 1 or its tip bending moment must be zero.

Now invoking the truncation assumption, the elastic deflection of each link is assumed to be described by a finite number of flexible modes, and subsequently the system is of only finite dimensions. For this truncated system, because it has been proven already that the largest invariant set $V = 0$ ($\dot{V} = 0$) is the final equilibrium position, the asymptotic stability directly follows the LaSalle’s theorem.

4 Simulation Tests

Numerical simulations are carried out on a two-link flexible smart materials robot moving in the horizontal plane. The system is modeled by FEM, system parameters are given in Table 1 where PM refers to piezoelectric material. In simulation, $\theta_{d1} = 30^\circ$ and $\theta_{d2} = 20^\circ$ while $\theta_1(0) = 0^\circ$ and $\theta_2(0) = 0^\circ$. The robot is assumed to be initially at rest without any deformation.

<table>
<thead>
<tr>
<th>Table 1: System Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length (m)</td>
</tr>
<tr>
<td>Beam width (m)</td>
</tr>
<tr>
<td>Sensor thickness (m)</td>
</tr>
<tr>
<td>Actuator thickness (m)</td>
</tr>
<tr>
<td>Beam flexural rigidity (N/m^2)</td>
</tr>
<tr>
<td>Beam linear density (Kg/m)</td>
</tr>
<tr>
<td>Beam stiffness (N/m^2)</td>
</tr>
<tr>
<td>PM length (m)</td>
</tr>
<tr>
<td>PM stiffness (N/m^2)</td>
</tr>
<tr>
<td>PM density (Kg/m^3)</td>
</tr>
<tr>
<td>Impermittivity (m/rod/s)</td>
</tr>
<tr>
<td>Coupling parameter (v/m)</td>
</tr>
<tr>
<td>Hub inertia (Kg/m^2)</td>
</tr>
<tr>
<td>Payload (Kg)</td>
</tr>
</tbody>
</table>

Following the discussion in [8], the PD controller be

$$\begin{align*}
\tau_{PD1} &= -21.6(\theta_1 - \theta_1^*) - 17.3\dot{\theta}_1 \\
\tau_{PD2} &= -14.0(\theta_2 - \theta_2^*) - 9.3\dot{\theta}_2
\end{align*}$$

The two energy based robust controllers be

**EBRC 1:** $\tau_i = \tau_{PD1} - k_{vi}w_i(L_i,t)\int_0^t \dot{\theta}_i(s)w_i(L_i,s)ds$

**EBRC 2:** $\tau_i = \tau_{PD1} - k_{vi}w_i''(0,t)\int_0^t \dot{\theta}_i(s)w_i''(0,s)ds$

where $w_i(L,t)$ are the tip deflections for each link, and $w_i''(0,t)$ are the base strain feedback for each link.

While the DMFC and CMFC controllers used for simulation study are given by

**DMFC:** $u_i = \begin{bmatrix} \tau_i \\ v_i \end{bmatrix} = \begin{bmatrix} \tau_{PD} \\ -k_{vi}I_i(t) \end{bmatrix}$

**CMFC 1:** $u_i = \begin{bmatrix} \tau_i \\ v_i \end{bmatrix} = \begin{bmatrix} \tau_{PD} - \tau_i \\ -k_{vi}I_i(t) \end{bmatrix}$

**CMFC 2:** $u_i = \begin{bmatrix} \tau_i \\ v_i \end{bmatrix} = \begin{bmatrix} \tau_{PD} - \tau_i \\ -k_{vi}I_i(t) \end{bmatrix}$

Figure 2 shows the deflection of DMFC with large enough feedback gain $k_{vi} = 6 \times 10^6$ due to the low authority of the piezoelectric material in comparison with the relatively good EBRC with $k_{vi} = 6000$, $k_{vi} = 2000$. It can be seen that DMFC is quite effective in residual vibration suppression in comparison to EBRC. Though the transient responses are about the same, there is a big difference at steady state. While there is no residual vibration at steady state for DMFC, there exists residual vibration of small magnitude for EBRC. Figure 3 shows the tip deflection of DMFC and CMFC, it can be seen that CMFC gave a bit quicker response than DMFC. For completeness, bounded joint angle and torque control of DMFC and CMFC are shown in Figure 4 and Figure 5.

Through the simulation study, we have shown the effectiveness of both DMFC and CMFC controllers despite its control low authority in control action. For more precise control performance, more pairs of piezoelectric actuators and sensors can be bonded. It should be pointed out that DMFC and CMFC can be applied according to different system configurations, and other kinds of feedbacks and/or other kinds of combinations of feedbacks can also be considered depending on the available sensor facilities, since the controllers actually allows a great deal of freedoms of engineering implementation and controller design.

5 Conclusion

In this paper, model-free regulation for multi-link flexible smart materials robot is presented. Theoretical proofs have shown that both of them can stabilize the closed-loop system, and the controllers are independent of system parameters and hence possess stability robustness to parameter variations. Furthermore the controllers can be easily implemented because the signals used in the feedback control can be measured directly or be chosen as measurable ones.
Numerical simulations have shown that system response converges fast and the residue vibrations are effectively suppressed using controllers proposed.

References


