Autonomous Vehicle Positioning with GPS in Urban Canyon Environments

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Abstract
The Global Positioning System (GPS) has been widely used in land vehicle navigation applications. However, the positioning systems based on GPS alone face great problems in the so called urban canyon environments where the GPS signals are often blocked by high buildings and there are not enough available satellites signals to estimate the positioning information of a fix. To solve the problem, a constrained method is presented by approximately modeling the path of the vehicle in the urban canyon environments as pieces of lines. By adding this constraint, the minimum number of available satellites reduces to two, which is satisfied in many urban canyon environments. Then, a state-augmentation method is proposed and the Extended Kalman Filtering (EKF) technique is adopted to simultaneously estimate the positions of the GPS receiver and the parameters of the line. Simulation results show that this approach can solve the urban canyon problems successfully.

1 Introduction
The Global Positioning System (GPS) is a worldwide, 24-hour, all weather three dimensional positioning system developed by the United States Department of Defense (DOD), which consists of 27 operational satellites. Originally developed by the DOD as a military system, GPS has become a global utility. It benefits users around the world in many different applications, including air, road, marine, and rail navigation, telecommunications, emergency response, oil exploration, mining, and many more. With the effect of the discontinuation of the Selective Availability (SA) on May 2, 2000 which intentionally degraded the civilian GPS signal, the improved, non-degraded signal will increase civilian accuracy by an order of magnitude, and have immediate implications in areas such as vehicle navigation.

To achieve better performance, differential GPS (DGPS) techniques are often used. The positioning performance for DGPS can achieve meter level or even centimeter level accuracy. However, the positioning system based on GPS or DGPS alone are challenged by urban canyon environments in terms of delivery of enough continuous positioning information. To obtain the positioning information of a fix, signals from at least four satellites are required. However, in urban areas, GPS signals are often blocked by high buildings (in the so called “urban canyons”). This means that for a significant percentage of the journey time a pointwise position solution is not available.

To tackle the problem associated with GPS in urban canyon environments, several approaches have been proposed. The most obvious way to alleviate such problems is to increase the number of visible satellites and enhance the geometry of the satellite coverage. In [2][8], the Russian Global Navigation Satellite System (GLONASS) is used to augment the GPS by offering more satellites in view and thus increasing the satellites availability. Advanced GPS receivers [9][10] have been developed which can access up to eight or more GPS satellites, and to thereby minimize any risk of urban blockage. Another approach is to find a constrained solution. For example, in marine-vehicle applications, the altitude could be defined as sea level [1]. In land vehicle applications, if the altitude changes slowly, it is often considered constant and assumed to be known [6]. The third approach is to using external references [1] such as an altimeter or a precise clock [7]. The fourth approach is to integrate the GPS with dead reckoning sensors such as INS and encoders to provide continuous positioning information [2][3][4][5]. Dead reckoning systems determine a vehicle’s location relative to an initial position by integrating measured increments and directions of travel, and essentially operate as an extrapolator of GPS positions [2]. When there are not enough GPS in view, dead reckoning systems give the positioning information, and when there are enough GPS satellite signals available, the positioning information is given by the GPS alone or the combination of the GPS and the dead reckon-
ing system. On the other hand, because the performance of a dead reckoning system degrades over time in an uncontrolled manner, the dead reckoning system is recalibrated by the GPS as the GPS solution is available \([2][3][4][5]\). The fourth approach to solve the urban canyon problem is modeling the dynamics and using a Kalman-filter-based approach \([1]\). The Kalman filter time update always provides a position estimate, even if no pseudorange measurements are available. But the covariance of this estimate will increase in at least one direction when the number of independent range measurements is less than four.

In this paper, another constrained solution is proposed to solve the problem by approximately modeling the path of the vehicle as pieces of curves such as straight lines, arcs, polynomials and so on. As the vehicle travels in an urban area, its path is always constrained in a certain pieces of road. By approximately modeling the path of the vehicle as a line resembling the road known \textit{a priori}, fewer GPS satellites are necessary to obtain the positioning information. In fact, the minimum number of the available satellites drops to two. Fortunately, detailed maps are usually available for most cities. Accordingly, the city map can be modeled as junctions connected by piecewise continuous lines. In such a manner, the information of the map is stored in the database of the proposed GPS positioning system.

Though an approximate model of the vehicle's path can be obtained from the map database, the actual path may deviate from the approximate model and are to be estimated. In this paper, the state augmentation method and the extended Kalman filtering (EKF) technique are used together to simultaneously estimate the parameters of the actual path and the positioning information.

The proposed approach has the following advantages: (1) using the new method for GPS positioning, the minimum number of available satellites drops to 2, instead of 4, which is satisfied in many urban canyon environments; (2) the new approach does not require any other sensors and thus keeps the positioning system cost low; and (3) by employing EKF, the proposed method can provide the positioning information and the parameters of the path simultaneously.

### 2 Problem Formulation

GPS positioning are often solved in the earth-center-earth-fixed (ECEF) rectangular coordinate system, where the user's position is denoted by \((x, y, z)\) and the \(i\)th satellite position is denoted by \((X_i, Y_i, Z_i)\) and \(i = 1, 2, \ldots, n_j\) with \(n_j\) being the number of the available satellites. The pseudorange measurement, \(\rho_i\), from the \(i\)th satellite to the user is given by

\[
\rho_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2 + B_r + v_i},
\]

\(i = 1, 2, \ldots, n_j\) (1)

where \(B_r\) is the distance corresponding to the user's clock bias with respect to the GPS time and \(v_i\) represents all the other errors contributed to the pseudorange measurements. If stand-alone GPS are used, \(v_i\) contains some common mode errors and thus are correlated to each other. If DGPS is adopted, the common mode errors in \(v_i\) are eliminated through differencing. Thus \(v_i\) contains only uncommon mode errors and can be considered as uncorrelated to each other. In this case, the error terms can be approximately characterized by zero-mean Gaussian white noise with typical standard deviation of 1 meter. Since the satellite positions can be precomputed from the ephemeris data, the user position and clock bias can be derived from (1) by neglecting \(v_i\). Since there are four unknowns, \(x, y, z\) and \(B_r\), in the pseudorange equations, at least four satellites are needed at each epoch.

When the vehicle with GPS receiver on board travels in urban areas, the GPS signals are often blocked by high buildings which form the so-called "urban canyons" environments. In such situations, if the number of available satellites is less than four at a given epoch, then the complete GPS solution cannot be obtained through the pseudorange equations (1) directly.

As mentioned in Section 1, several approaches have been proposed to tackle the problem associated with GPS in urban canyon environments. In this paper, another constrained solution is provided to solve the problem by approximately modeling the path of the vehicle by pieces of curves in the urban canyon environments. As the vehicle travels in an urban area, its path is always confined in a certain piece of road. By modeling the path of the vehicle as a curve resembling the shape of the road, fewer GPS satellites are necessary to obtain the positioning information.

Generally, a curve in space can be regarded as the intersection of two surfaces. Recall that any surface in space can be described by

\[ S_1(x, y, z) = 0 \]

Therefore, a curve can be generally modeled by

\[
\begin{align*}
S_1(x, y, z) &= 0 \\
S_2(x, y, z) &= 0
\end{align*}
\]

Combining (1) and the two equations in the general line model (2) leads to \(n_j + 2\) equations at each epoch with four unknowns, the current position \((x, y, z)\) and the user's clock bias \(B_r\), provided that the parameters of the line are known \textit{a priori}. To obtain the four unknowns at each epoch, the number of equations cannot be less than 4, i.e. \(n_j \geq 2\). Thus, the minimum number of satellites to obtain the position and clock bias information drops to 2.

In order to acquire high accuracy of positioning, DGPS are employed instead of stand-alone GPS. In this paper, the Kalman filtering techniques are used
to further improve the accuracy of the position estimates. The operations of the overall DGPS-based positioning system are illustrated in Figure 1. When the GPS receiver is able to access enough satellite signals with good geometry, the positioning information will be estimated through traditional approach by using the pseudorange equation (1) only. If the vehicle travels in urban canyon environments, some GPS signals are lost by blockage. In this case, the positioning system estimates the positioning information by utilizing both the available GPS signals (suppose that there are at least two GPS satellites in view) and the model of the path of the vehicle.

![Figure 1: Flowchart of the positioning system.](image)

3  EKF For Position Estimation

Because the pseudorange equation (1) is inherently nonlinear, the EKF technique is adopted to estimate the positioning information. EKF is a form of the Kalman filter “extended” to nonlinear dynamic systems. The most important features of EKF are that it uses both the nonlinear and linearized models of the system, and relinearizes the nonlinear system about each new estimate as it becomes available. First, EKF solution for the case where there are enough GPS satellites are discussed.

3.1 In Non-Urban-Canyon Area

When the receiver is in low dynamic motion, (i.e., near constant velocity), the velocity is modeled as a random-walk process, positions are models as the integral of velocity. Physically the clock bias develops as the integral of the frequency error of the receiver clock oscillator [1]. Thus, the receiver clock bias can be described by a two-state model

\[
\begin{bmatrix}
\dot{B}_w(t) \\
\dot{B}_r(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
B_w(t) \\
B_r(t)
\end{bmatrix} +
\begin{bmatrix}
w_\phi(t) \\
w_f(t)
\end{bmatrix}
\]

where \(w_\phi\) and \(w_f\) are the process noise driving the phase and the frequency error states, respectively. Then, the state-space model for the GPS receiver is given by

\[
x = Fx + w
\]

where

\[
x = \begin{bmatrix} x & y & z & B_r & \dot{x} & \dot{y} & \dot{z} & \dot{B}_r \end{bmatrix}^T
\]

\[
F = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
w = \begin{bmatrix}
w_{B_w} & w_{\dot{x}} & w_{\dot{y}} & w_{\dot{z}} & w_{\dot{B}_r} \end{bmatrix}^T
\]

The \(w_v\) terms would be selected to model the random variations in the velocity. Obviously, this state-space model is linear. The pseudorange equation (1) is regarded as the measurement equation, which can be rewritten as

\[
\rho = h(x) + v
\]

where

\[
\rho = \begin{bmatrix} \rho_1 & \cdots & \rho_m \end{bmatrix}^T
\]

\[
h(x) = \begin{bmatrix}
\sqrt{(x - X_1)^2 + (y - Y_1)^2 + (z - Z_1)^2 + B_r} \\
\vdots \\
\sqrt{(x - X_m)^2 + (y - Y_m)^2 + (z - Z_m)^2 + B_r}
\end{bmatrix}
\]

\[
v = \begin{bmatrix} v_1 & \cdots & v_m \end{bmatrix}^T
\]

(4) and (5) are continuous-time model for the system. The corresponding discrete-time model is described by

\[
x(k + 1) = \Phi x(k) + w_d(k)
\]

\[
\rho(k) = h(x(k)) + v(k)
\]

(6) and (7) are continuous-time model for the system. The corresponding discrete-time model is described by

\[
x(k + 1) = \Phi x(k) + w_d(k)
\]

\[
\rho(k) = h(x(k)) + v(k)
\]

where \(\Phi = e^{FT}\), with \(T_s\) being the sampling period, and \(w_d(k)\) is a discrete-time white Gaussian sequence that is statistically equivalent through its first two moments to \(\int_{t_k}^{t_{k+1}} f \Phi w(t) dt\) [11].

Under the assumption that the noises, \(w_d\) and \(v\), are zero-mean normal distributed white noises and are characterized by covariance matrices, \(Q_d\) and \(R\), respectively, the EKF equations are given by

\[
\bar{x}^-(k) = \Phi \bar{x}^+(k - 1)
\]

\[
P^-(k) = \Phi P^+(k - 1) \Phi^T + Q_d
\]

\[
K(k) = P^-(k) H^T(k) [H(k) P^-(k) H^T(k) + R(k)]^{-1}
\]

\[
P^+(k) = [I - K(k) H(k)] P^-(k)
\]

\[
\bar{x}^+(k) = \bar{x}^-(k) + K(k) [\rho(k) - h(x^-(k))]
\]
where

$$\mathbf{H}(k) = \frac{\partial h(x)}{\partial x} \bigg|_{x=x_{-(k)}} = \begin{bmatrix} H_1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ H_m & 0 & 0 & 0 \end{bmatrix}$$

with

$$H_i = \begin{bmatrix} \frac{\partial p_i}{\partial x} & \frac{\partial p_i}{\partial y} & \frac{\partial p_i}{\partial z} & 1 \end{bmatrix}^{T} \bigg|_{x=x_{-(k)}}$$

and

$$\frac{\partial p_i}{\partial x} = \frac{x - X_i}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}}$$
$$\frac{\partial p_i}{\partial y} = \frac{y - Y_i}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}}$$
$$\frac{\partial p_i}{\partial z} = \frac{z - Z_i}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}}$$

3.2 In Urban Canyon Environment

If the curve model of the path of the vehicle is known, the positioning information of the vehicle can be calculated from the two (or more) pseudorange equations and the curve model. Notice that in reality, most pieces of roads are straight lines, arcs, or other simple smooth curves. Therefore, without loosening generality, assume that both y and z can be written as an explicit function of x. Then, the roads can be simply modeled by

$$\begin{cases} y = f_1(x) \\ z = f_2(x) \end{cases}$$

(13)

Thus, for the case of two available satellites, the set of equations to be solved are

$$\rho_1 = \rho_{10} + B_r + v_1$$
$$\rho_2 = \rho_{20} + B_r + v_2$$
$$y = f_1(x)$$
$$z = f_2(x)$$

(14) (15) (16) (17)

In real implementation, though the mathematical model for a certain road can be achieved from the map, the actual path of the vehicle may deviate from this model. Therefore, in real implementation, a model for the path of the vehicle is established which resembles the form of the road model but whose parameters are unknown and to be estimated. In this paper, the state augmentation method and the EKF technique are used to together to estimate the parameters of the line and the positioning information simultaneously.

The fundamental concept of state augmentation is to treat the unknown parameters of the curve also as states. The curve of the vehicle’s path with unknown parameters can be written as

$$\begin{cases} y = f_1(x, \mathbf{p}_c) \\ z = f_2(x, \mathbf{p}_c) \end{cases}$$

(18)

where $\mathbf{p}_c = [p_1, p_2, \ldots, p_l]^T$ denotes the unknown parameters of the curve and l is the number of the parameters to be estimated.

The resulting state-space model is given by

$$\dot{x}_1 = F_1 x_1 + w_1$$

(19)

where

$$x_1 = \begin{bmatrix} x & \dot{x} & \ddot{x} & \rho_{10} & \rho_{20} & \ldots & \rho_{1l} & \rho_{2l} \end{bmatrix}^T \in \mathbb{R}^{4+l}$$
$$F_1 = \begin{bmatrix} F_{11} & 0 \times l \\ 0 \times 4 & 0 \times l \end{bmatrix} \in \mathbb{R}^{(4+l) \times (4+l)}$$
$$w_1 = \begin{bmatrix} 0 & w_{10} & \ldots & w_{1l} \end{bmatrix}^T \in \mathbb{R}^{l \times 1}$$

with $F_{11} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \end{bmatrix}$.

According to (1), the measurement equation can be written as

$$\mathbf{h}_1(x) = h_1(x_1(t)) + \mathbf{v}(t)$$

(20)

where

$$\mathbf{h}_1(x) = [h_{1i}] \in \mathbb{R}^m$$

with $h_{1i} = \rho_{1i} + B_r$.

The EKF equations to estimate $x_1$ are similar to (8)-(12). Note that the measurement matrix becomes $\mathbf{H}_1(k) = [\mathbf{H}_{1i}]|_{x_1=x^*_{-k}} \in \mathbb{R}^{m \times (4+l)}$ with

$$H_{11i} = \frac{1}{\rho_{10}} \left[ (f_1(x, \mathbf{p}) - Y_i) \frac{\partial f_1(x, \mathbf{p})}{\partial x} \right]$$
$$H_{12i} = 1, \ H_{13i} = 0, \ H_{14i} = 0$$
$$H_{1ij} = \frac{1}{\rho_{10}} \left[ (f_1(x, \mathbf{p}) - Y_i) \frac{\partial f_1(x, \mathbf{p})}{\partial p_j} \right]$$

For example, assume that the vehicle moves along a straight road which can be approximately modeled as

$$\begin{cases} y = p_{10}x + p_{20} \\ z = p_{30}x + p_{40} \end{cases}$$

(21)

where $p_{10}$, $p_{20}$, $p_{30}$ and $p_{40}$ are four known parameters. Without loosening generality, assume that this straight line models the centerline of the road. As the vehicles travels along this road, its path may not coincide with the centerline of the road. Hence we shall assume that its path is still a straight line with the same form but slightly different parameters, i.e.,

$$\begin{cases} y = p_{1x} + p_{2} \\ z = p_{3x} + p_{4} \end{cases}$$

(22)

where $p_1$, $p_2$, $p_3$ and $p_4$ are four unknown parameters to be estimated. The algorithm for joint parameter and state estimation follows the above analysis.
where the augmented state is given by

\[
x_1 = \begin{bmatrix} x & B_r & \dot{x} & \dot{B}_r & p_1 & p_2 & p_3 & p_4 \end{bmatrix}^T \tag{23}
\]

The EKF algorithm follows equations (8)-(12), with

\[
H_i(k) = \frac{\partial h_i(x_1)}{\partial x_1} = [H_{ij}]x_1 \in \mathbb{R}^{m \times 8}
\]

where

\[H_{1i1} = \frac{1}{\rho_{i0}} \left[ x - X_i + p_1(p_1x + p_2 - Y_i) + p_3(x_3x + p_4 - Z_i) \right],\]

\[H_{1i2} = 1, \quad H_{1i3} = 0, \quad H_{1i4} = 0,\]

\[H_{1i5} = \frac{1}{\rho_{i0}} [x(p_1x + p_2 - Y_i)],\]

\[H_{1i6} = \frac{1}{\rho_{i0}} (p_1x + p_2 - Y_i),\]

\[H_{1i7} = \frac{1}{\rho_{i0}} [x(p_3x + p_4 - Z_i)],\]

\[H_{1i8} = \frac{1}{\rho_{i0}} (x_3x + p_4 - Z_i).\]

In the EKF process, the nonlinear system is linearized about each new estimate as it becomes available. Thus the EKF is designed to work well as long as the estimates are near their true values. Therefore, the initial states of the augmented system cannot be given arbitrarily. They must not deviate from their true values too much. Fortunately, since the detailed digital map of the city is available, the curve model for each pieces of road can be determined prior to the navigation process. Hence the parameters of the curve model for the road can be regarded as the initial estimates of the parameters of the curve modeling the vehicle's path.

4 Simulation Studies

In this section, the effectiveness of the proposed method is demonstrated through computer simulations. Assume that the vehicle equipped with GPS receiver travels somewhere with local horizontal plane coordinate system originated at latitude \(\lambda = 1^\circ 23' 15.9''\), longitude \(\phi = 103^\circ 56' 32.3''\), and height \(h = 0\) m in the ECEF coordinate frame. The road map of the navigation environment with respect to the local horizontal plane frame is shown in Figure 2, where the vehicle travels along the path A-B-C-D-E-F-A. The roads AB, CD and EF are straight roads, the roads BC and EF are polynomial roads, and the road DE is a descending arc road. All the parameters for modeling the roads are known \textit{a priori}.

The urban canyon environments along these roads are annotated by dashed lines where only two or three satellites are in view.

To compare the performances of the proposed joint parameter and state estimation approach and the original EKF method which utilizes the system model (4) and (5) all the time for GPS positioning in urban canyon environments. Figure 3 shows the positioning distance errors in case of using the conventional EKF method without path modeling. It is obvious that due to the use of DGPS, EKF method, and by using all the satellite signals available, the positioning errors in non-urban-canyon areas are very small, in the meter level. However, in the urban canyon environments along the path, the positioning errors using the original approach without modeling the straight path increases fast and become intolerable. As shown in Figure 3, during the time intervals \(t \in [250, 340], [550, 600], [770, 880], [970, 1060]\) and \([1160, 1280]\), the vehicle is in urban canyon environments, where the distance errors grow very fast. In contrast, the proposed new method by combining the straight line model generates much lower positioning errors, as shown in Figure 4. The distance errors are within 3 meters, which is acceptable in ordinary vehicle navigation applications. The simulation results show that the proposed method for GPS positioning in urban canyon environments is feasible and effective.

5 Conclusion

The positioning system based on GPS or DGPS alone are challenged by urban canyon environments in terms of delivery of enough positioning information. To obtain the positioning information of a fix, signals from at least four satellites are required. However, in urban areas, GPS signals are often blocked by high buildings. This means that for a significant percentage of the journey time a point-wise position solution is not available. In this paper, a constrained method is presented by approximately modeling the path of the vehicle in the urban canyon environments as pieces of lines. By adding this constraint to the set of pseudorange equations, the min-
minimum number of available satellites reduces to two, which is satisfied in many urban canyon environments. In this paper, a state augmentation method is also presented to estimate the parameters of the line and the positioning information simultaneously. Based on the augmented system, EKF is employed to estimate the states. The most significant advantages of this method are that it can successfully solve the urban canyon problem associated with GPS, and it does not involve any other sensors and thus keeps the cost of the positioning system low.

References


