Multi-Degree-of-Freedom Spherical Permanent Magnet Motors

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Abstract

The paper describes the analysis, design and control of two new versions of spherical permanent magnet motor, which are capable of two and three degrees-of-freedom of controlled motion, and have a high specific torque capability. Their magnetic field distribution is established using an analytical technique formulated in spherical coordinates. This enables the torque vector and back-emf to be derived in closed forms. An optimal design procedure, to achieve maximum output torque or maximum acceleration for a given payload, is presented. The control of the motors, whose dynamics are similar to those of robotic manipulators, is facilitated by the establishment of a complete actuation system model and the application of the PD or computed torque control law. The validity of the analysis and design techniques, and the effectiveness of the control strategy, are confirmed by measurements.

1. Introduction

Recent advances in robotics, office automation, and intelligent flexible manufacturing and assembly systems have necessitated the further development of programmable, servo-controlled, high speed actuators with multiple degrees-of-freedom. Such devices might, for example, be appropriate for a robotic wrist, which normally requires three degrees-of-freedom motion in yaw, pitch and roll in order to position an end-effector at a specified orientation. The performance requirements for the wrist usually include speed, accuracy, and dexterity, as well as volumetric efficiency. At present, the wrist motion is realised almost exclusively by using a separate motor/actuator for each axis. A typical example is the Puma 3-axis wrist shown in Fig. 1 [1] in which three motors, together with a relatively complicated and heavy transmission system, are used to deliver the required motion in the three axes. This arrangement inevitably compromises the dynamic performance and servo-tracking accuracy, due to the combined effects of inertia, backlash, non-linear friction, and elastic deformation of gears. Motors with multiple degrees-of-freedom should alleviate these problems, whilst being lighter and more efficient.

Fig. 1 Schematic of Puma 3-axis wrist

Multi-degree-of-freedom motors have been the subject of research for several decades. For example, a spherical induction motor [2] was first proposed by Laithwaite et al., and its electromagnetic field was later analysed by Davey et al. [3] using an idealised distributed current sheet model. However, this form of actuator remains uncommercialised, probably due to its relatively complex stator core and winding arrangement, and the inherently poor servo characteristics of induction motors. Spherical motors based on the variable reluctance stepper motor principle were proposed by Lee et al. [4]-[5]. These also had a relatively complex stator and rotor structure, and, to date, no measured results have been reported. A spherical permanent magnet motor was first reported by Kaneko et al. [6], and variations were subsequently proposed by Wallace et al. [7] and Andresen [8]. However, since they all used a complicated three-axis gimbal system they are not mechanically robust, and have a relatively low specific torque capability, due to the large airgap which is...
necessary to accommodate the gimbal system. Further, the three dimensional nature of the electromagnetic field distribution in all the foregoing spherical motors makes their electromagnetic and dynamic behaviour difficult to analyse, and this has been a significant obstacle to their design optimisation and servo application. As a result, the potential benefits of employing spherical motors have not been realised.

This paper describes two new forms of spherical permanent magnet motors, and highlights the key design and operational issues.

2. Motor topologies

2.1 Two DOF version

Fig. 2 shows a schematic of the spherical motor which is capable of two degrees-of-freedom. It has a spherical permanent magnet rotor and a simple 3-phase stator coil arrangement, the rotor bearing being a low friction surface coating. Accommodated on the stator are three orthogonal pairs of coils, which may be enclosed by an outer spherical iron shell in order to increase the flux-linkage and specific torque capability. The two-pole rare-earth permanent magnet rotor may be either solid or hollow or have an inner spherical iron core, and can be either diametrically or radially magnetised. On the application of current to the stator coils, the resulting torque will orientate the rotor so as to minimise the system potential energy. Thus, control of the rotor orientation is achieved by varying the winding currents. Since the magnetic field produced by the permanent magnet rotor is axially symmetric, the actuator has only two independently controllable torque components.

2.2 Three DOF version

Fig. 3 shows a schematic of the three DOF spherical motor. It differs from the two DOF version in that it has a four-pole spherical permanent magnet rotor, which is formed from two pairs of parallel magnetised quarter-spheres, whilst the stator has four sets of windings, which are arranged so that three independently controllable torque components can be developed by energising the appropriate windings. As with the 2 DOF version, the stator can be either air-cored or iron-cored by enclosing the windings with a spherical iron shell.

3. Analysis and design

For both motor topologies, the magnetic field distribution produced by the spherical magnet rotor can be established analytically in the spherical co-ordinate system shown in Fig. 4. In general, the flux density distribution consists of a series of spherical harmonics with a dominant fundamental. For the 2-pole diametrically magnetised rotor employed in the 2 DOF version, however, the flux density components are axially symmetric and sinusoidally distributed with respect to $\theta$.
Fig. 5 compares the analytically predicted flux density with finite element calculations for both the 2 DOF and 3 DOF motors.

![Graph](image1)

Fig. 5 Radial component of flux density as a function of (a) $\theta$, and (b) $\alpha$ in a 3 DOF motor.

The torque exerted on the rotor, resulting from the interaction between the current in a stator coil shown in Fig. 6, and the rotor magnetic field, is given by:

$$ T = -\int_V r \times (J \times B) dV \quad (1) $$

which can be reduced to a simple form. For example [9], for the 3 DOF actuator, the result of equation (1), neglecting high order harmonics, is given by:

$$ T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = T_m \begin{bmatrix} v_x^2 - v_y^2 \\ -v_x v_y \\ v_x v_z \end{bmatrix} \quad (2) $$

where $\mathbf{v} = [v_x, v_y, v_z]^T$ is the direction cosines of the winding axes. The torque magnitude, $T_m$, is related to the magnet remanence $B_{rem}$, the winding current density $J$, and the rotor and stator radii $R_m$ and $R_s$ as well as the winding geometric angles $\delta_0$ and $\delta_1$, and is given by:

$$ T_m = \frac{15\pi \sqrt{2} B_{rem} I R_m^4}{8(2\mu_r + 3)} \ln\left(\frac{R_s}{R_0}\right) \left(\sin^3 \delta_1 - \sin^3 \delta_0\right) \quad (3) $$

For the two DOF motor, the torque magnitude $T_m$ is also dependent upon $B_{rem}$ and $J$, as well as the geometrical parameters of the rotor and the winding. If the airgap between the rotor and winding is $G$, then for given values of $B_{rem}$, $J$, $\delta_0$, $\delta_1$, and $R_s$, $T_m$ is a function of $R_m/R_s$. This relation is plotted in Fig. 7 for both the 2 DOF and 3 DOF motor.

![Graph](image2)

Fig. 6 Winding distribution

(a) 2 DOF ($B_{rem}=1.2T, J=2.0A/mm^2, R_s = 0.044m, \delta_0=70^\circ, \delta_1=35.570^\circ$)

![Graph](image3)

(b) 3 DOF ($B_{rem}=1.2T, J=2.0A/mm^2, R_s = 0.042m, \delta_0=50^\circ, \delta_1=30^\circ$)

Fig. 7 Torque vs. $R_m/R_s$ curves
The total torque of each motor is the vector sum of the torque components due to the individual windings. By a similar integration procedure, the flux-linkage of a pair of windings due to the rotor magnetic field can be formulated and the back-emf subsequently derived.

The design of the motors can be optimised with respect to a given criterion. For example, if the maximum torque capability is required, then as shown in Fig. 7, there is an optimal ratio of $R_m/R_s$ that yields maximum torque. When the effect of Coulomb friction is taken into account, the net output torque $T_{eff}$ is reduced. As a result, the optimal value of the ratio $x_r = R_m/R_s$ is also reduced as shown in Fig. 8 for the 2 DOF motor.

![Fig. 8 Torque vs. $R_m/R_s$ curves (2 DOF motor)](image)

In many applications, a common requirement is for maximum acceleration from a motor, so as to achieve the fastest dynamic response for a given payload. Assuming that the payload can be approximated by a point mass, then its dynamic effect can be taken into account to establish the system acceleration as a function of the design parameters. By way of example, Fig. 9 shows the maximum attainable acceleration of the 2 DOF motor as a function of $R_s$ and $R_m/R_s$.

![Fig. 9 Maximum acceleration of 2 DOF motor as a function of $R_s$ and $R_m/R_s$](image)

Based on the above results, an integrated design procedure can be formulated to yield optimal designs in terms of a chosen performance criterion for a given specification.

### 4. Rotor position sensing

For closed-loop control, it is necessary to have position feedback signals. For the 2 DOF motor employing a diametrically magnetised rotor, the orientation of the rotor, represented by a set of Euler angles $[\beta \alpha \phi]$, may be deduced by simply using four sensors, either Hall effect or magnetoresistive devices, for example, positioned as shown in Fig. 10.

![Fig. 10 Sensor arrangement for 2 DOF motor](image)

Sensors 1 and 2 are deployed symmetrically with respect to the $x$ axis on the $x$-$y$ plane, and sensors 3 and 4 are similarly deployed on the $x$-$z$ plane. The magnetic field which is measured by the sensors is the combination of the radial components produced by all the field sources, viz., the rotor magnet and the coil currents, $i_A$, $i_B$ and $i_C$. Due to the symmetry of the field and sensor system, $\alpha$ and $\beta$ can be deduced from the measurement of the sensor outputs $V_{sj}$ ($j = 1, \ldots, 4$) and the coil currents $i_B$ and $i_C$ by the following equations:

$$\alpha = \sin^{-1} \left[ \frac{V_{s2} - V_{s1} - 2k_{ib}i_B}{2kB_{ro}\sin\alpha_0} \right]$$  \hspace{1cm} (4)

$$\beta = \sin^{-1} \left[ \frac{V_{s3} - V_{s4} - 2k_{ic}i_C}{2kB_{ro}\sin\alpha_0\cos\alpha} \right]$$  \hspace{1cm} (5)

where $k$ is the sensitivity of the sensors, and $k_{ib}$ and $k_{ic}$ are coefficients relating the sensor outputs to the coil currents $i_B$ and $i_C$. For the 3 DOF motor, the position sensing mechanism could similarly be arranged. However, in this case, more sensors are required to deduce the complete set of Euler angles $[\alpha \beta \phi]$. The situation is further compounded by the existence of higher order field harmonics and the need to eliminate ambiguities arising from the 4-pole rotor magnetisation.
5. Control of spherical motors

A generalised dynamic model for both the 2 DOF and 3 DOF motors is given by:

\[
\begin{align*}
MQ\ddot{Q} & + CQ\dot{Q} + G + \tau_f = \tau_M = \tau_{ET}i_w \\
Li_w + Ri_w - K_{ET}^TQ_E &= u_E \\
\end{align*}
\]

where \(Q_E\) is the Euler angle vector representing the rotor orientation, and \(M, C\) and \(G\) denote the inertia matrix, the Coriolis and centripetal force matrix, and the gravitational torque vector, respectively. \(u_E\) is the coil terminal voltage vector, \(i_w\) is the coil current vector, \(L\) is the coil inductance matrix, \(R\) is the diagonal coil resistance matrix, \(\tau_f\) is the vector representing the Coulomb and viscous friction, and \(K_{ET}\) is defined as the motor torque matrix. It can be shown that, in non-singular regions, eqn. (6) constitutes a Hamiltonian system, and, therefore, possesses a well-understood structure and similar important properties as the dynamic equations for robotic manipulators [10]. As a result, any advanced control law which is used for the control of robotic manipulators can be applied to the spherical motors.

As an example, a robust outer PD position control law [11] in conjunction with an inner PI current control law, as shown in Fig. 11, has been utilised for the control of the spherical motors. The role of the inner current tracking loop is to minimise the effects of the back-emf and current transients on the outer position servo loop, so that a robust design philosophy [11] can be used to determine the control gain matrices \(K_i\) and \(\Lambda\). In both the 2 DOF and 3 DOF motors, there exists a redundant control input which may be used for optimal control, e.g., to minimise the total energy consumption for a given torque demand. This control strategy is implemented by taking the weighted pseudo inverse of \(K_{ET}\), as denoted by \(K_{ET}^+\) in Fig. 11.

6. Prototypes and experimental results

Fig. 12 shows a prototype air-cored 2 DOF spherical motor, whose specification is given in Table 1. The motor is optimally designed to yield maximum acceleration when the rotor carries a miniature electronic camera as its payload. The winding angles were chosen such that each pair of windings has an identical torque capability.

![Prototype of 2 DOF spherical motor](image)

**Table 1 Specification of prototype 2 DOF spherical motor**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage</td>
<td>(\pm) 24 [V]</td>
</tr>
<tr>
<td>Torque</td>
<td>0.15 [Nm] continuous 0.40 [Nm] peak</td>
</tr>
<tr>
<td>Pan and tilt excursions</td>
<td>(\pm) 45(^\circ)</td>
</tr>
<tr>
<td>Nominal payload</td>
<td>(m_c = 0.05) [kg]; (l_c = 0.017) [m]</td>
</tr>
<tr>
<td>Max. angular Acc.</td>
<td>1540 [rad/s(^2)] no load, 423.6 [rad/s(^2)] with payload</td>
</tr>
<tr>
<td>Outer stator radius</td>
<td>0.044 [m]</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>0.024 [m]</td>
</tr>
</tbody>
</table>

The spherical rotor is a diametrically magnetised sintered NdFeB magnet with \(B_r = 1.25\) [T] and \(\mu_r = 1.07\).
The back-emf or torque constant of the motor was obtained by measuring the flux-linkage of each phase winding as a function of the rotor orientation, using an integrating flux-meter and four differentially connected rotor position sensors, as described in section 4. The measured and predicted flux-linkages of each winding are compared in Fig. 13. From these results, the back-emf or torque constant of each winding can be obtained using the least square curve-fitting technique, and is given in Table 2.

![Fig. 13 Winding flux-linkage as a function of \( \theta \)](image)

### Table 2 Measured and predicted back-emf or torque constant

<table>
<thead>
<tr>
<th>Winding</th>
<th>Measured</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.289</td>
<td>0.282</td>
</tr>
<tr>
<td>B</td>
<td>0.269</td>
<td>0.287</td>
</tr>
<tr>
<td>C</td>
<td>0.301</td>
<td>0.287</td>
</tr>
</tbody>
</table>

The control strategy described in section 5 has been implemented on the spherical motor system with a 1 [ms] sampling interval. The two Euler angles, which represent the rotor orientation, were obtained from differentially connected pairs of Hall sensors. An auto-calibration procedure was also incorporated into the control software to reduce temperature dependent effects on the measurement accuracy.

The controller gain matrices are given by 

\[
K_P = \text{diag}[0.5, 0.5], \quad A = \text{diag}[20, 20], \quad \text{and the weighting matrix } W \text{ was chosen to achieve minimum power consumption for a given torque demand. Since initial tests indicated that friction torque was greater than expected, due to manufacturing tolerances, an integral control action with anti-winding-up protection was added to the PD control law in order to improve the steady-state tracking accuracy.}

Fig. 14 shows a typical system step response.

![Fig. 14 Typical Euler angle response](image)

As is seen, the motor reaches its target orientation in less than 0.2 seconds, with no overshoot and almost zero steady-state error. Fig. 15 shows the tracking response to continuous exponential input demands given by:

\[
\beta = 0.3(1 - e^{-0.5t}) \quad ; \quad \alpha = 0.3(1 - e^{-0.5t^2})
\]

This demonstrates that the system has excellent trajectory tracking capability.

![Fig. 15 Euler angle response to exponential demand](image)

A prototype air-cored 3DOF motor, whose specification is given Table 3, has also been prototyped, Fig. 16.

### Table 3 Specification of prototype 3 DOF spherical motor

<table>
<thead>
<tr>
<th>Supply voltage</th>
<th>( \pm 24 \text{ [V]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque</td>
<td>0.25 [Nm] continuous 0.6 [Nm] peak</td>
</tr>
<tr>
<td>Pan and tilt excursions</td>
<td>( \pm 45^0 )</td>
</tr>
<tr>
<td>Continuous rotation</td>
<td>( \pm 360^0 )</td>
</tr>
<tr>
<td>Nominal payload</td>
<td>( m_c = 0.05 \text{[kg]} ); ( l_c = 0.017 \text{[m]} )</td>
</tr>
<tr>
<td>Max. angular Acc.</td>
<td>( 628 \text{[rad/s^2]} ) no load , ( 403 \text{[rad/s^2]} ) with payload</td>
</tr>
<tr>
<td>Outer stator radius</td>
<td>0.06 [m]</td>
</tr>
<tr>
<td>Rotor radius</td>
<td>0.031 [m]</td>
</tr>
</tbody>
</table>

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The actuator is optimally designed to yield maximum acceleration with the same payload as that of the 2 DOF motor. Fig. 17 compares the measured and predicted winding flux-linkage.

The control strategy described in section 5 has been applied to the spherical motor system with a 2 [ms] sampling interval. In this case, however, the PD control law is replaced by the computed torque control law [9]. The three Euler angles, which represent the rotor orientation, were obtained from the six Hall sensors. An auto-calibration procedure was also incorporated into the control software to reduce temperature dependent effects on the measurement accuracy. The controller gain matrices are given by $K_I = \text{diag}[20 \ 20 \ 20]$, $K_p = \text{diag}[800 \ 800 \ 800]$, and the weighting matrix $W$ was chosen to achieve minimum power consumption for a given torque demand.

Fig. 17 shows typical system step responses. Due to manufacturing tolerances, both the rotor and stator housing are not perfect spheres. This results in a significant amount of non-uniformly distributed stick-slip friction torque. The effects of the destabilising torque on the step responses are clearly visible. At the beginning of the transient, the rotor remains stationary until the torque is sufficiently large to overcome the break-away friction torque. Once the rotor starts to move, the friction torque decreases, thereby causing a certain amount of overshooting. Since the distribution of the non-linear friction torque is not uniform within the working envelope of the motor, complete decoupling between the Euler angles will be difficult to achieve using the computed torque control law. Consequently, a certain amount of coupling exists, as can be observed in the step responses in Fig. 18. Nevertheless, these results demonstrate the operation of the spherical motor. However, the performance of the system can be improved significantly by exerting tighter control on manufacturing tolerances or using an alternative bearing technology, such as an air bearing.
Fig. 18 Euler angle responses to step demands

(b) \( Q_\alpha = [0 \ 0.3 \ -0.3] \) (rad)

Fig. 19 3-DOF motor with slotted iron stator

Nevertheless a 3-DOF motor with slotted iron stator, Fig. 19, is being developed with the aid of magnetostatic finite element analyses to predict coil flux-linkage/rotor orientation data and assist in the investigation of alternative control strategies and the estimation of detent torque.

8. Conclusions

Two new forms of spherical motor, which are capable of two and three degrees-of-freedom, have been described. Each has a simple topology which enables it to be characterised analytically. They have a high specific torque, a robust mechanical structure, and a simple position sensing system. A complete kinematic and dynamic model of the motors has been established, a design methodology to achieve maximum output torque or maximum acceleration has been developed, and a control strategy for their closed-loop control has been described. The stability and performance of the control strategy is guaranteed through the properties of its dynamic equations. The validity and effectiveness of the developed analysis, and the design and control methodologies have been demonstrated on prototype air-cored 2-DOF and 3-DOF motors, and their associated control systems.

References