Abstract

This paper addresses two major issues in fixture layout design: (1) to determine the feasible fixture configurations that satisfy fundamental requirements such as kinematic localization and total fixturing (form-closure) and (2) to evaluate the acceptable fixture designs on several quality criteria and select the optimal fixture appropriate with practical demands. The performance objectives considered include the workpiece localization accuracy, and the norm and distribution of the locator contact forces. An efficient automated tool based on an interchange algorithm is developed for designing optimal fixture layout for arbitrary 3D parts. A thorough analysis is performed on the fixture characteristics during the single and multiple-criteria optimization process for different frequent cases, and on the inter-relationship between locators and clamps, leading to conclusions and strategies for performing fixture synthesis.

1. Introduction

Proper fixture design is crucial to product quality in terms of precision and accuracy in part fabrication and assembly. Fixturing systems, usually consisting of clamps and locators, must be capable to assure certain quality performances, besides of positioning and holding the workpiece throughout all the machining operations. Although there are a few design guidelines such as 3-2-1 rule, automated systems for designing fixtures based on CAD models have been slow to evolve.

This article describes a research approach to automated design of a class of fixtures for 3D workpieces. The parts considered to be fixtured present an arbitrary complex geometry, and the designed fixtures are limited to the minimum number of elements required, i.e. six locators and a clamp. Furthermore, the fixels are modeled as non-frictional point contacts and are restricted to be applied within a given collection of discrete candidate locations. In general, the set of fixture locations available is assumed to be a potentially very large collection; for example, the locations might be generated by discretizing the exterior surfaces of the workpiece. The goal of the fixture design is to determine first, from the proposed discrete domain, the feasible fixture configurations that satisfy the form-closure constraint. Secondly, the sets of acceptable fixture designs are evaluated on several criteria and optimal fixtures are selected. The performance measures considered in this work are the localization accuracy, and the norm and distribution of the locator contact forces. These objectives cover the most critical error sources encountered in a fixture design, the position errors and the unwanted stress in the part-fixture elements due to an overloaded or unbalanced force system.

The optimal fixture design approach is based on a concept of optimum experiment design. The automated developed algorithm enumerates efficiently the admissible designs exploiting the part geometry and performing a force analysis, and selects further the optimal fixture design that will assure a certain desired quality.

2. Related Work

Literature on general fixturing techniques is substantial, e.g., [1]. The essential requirement of fixturing is the century-old concept of form closure [2], which has been extensively studied in the field of robotics in recent years [3, 4]. There are several formal methods for analyzing performance of a given fixture based on the popular screw theory, dealing with issues such as kinematic closure [5], contact types and friction effects [6].

A different analysis approach based on the geometric perturbation technique was reported in [7]. An automatic modular fixture design procedure based on this method was developed in [8] to include geometric access constraints in addition to kinematic closure. The problem of designing modular fixtures gained more attention lately [9]. There has also been extensive research in fixture designs, focusing on workpiece and fixture structural rigidity [6], tool accessibility and path clearance [7].

The problem of fixture synthesis has been largely studied for the case of a fixed number of fixture elements (or fixels) [8, 10], particularly in the application to robotic manipulation and grasping for its obvious reasons [3, 4].

This article aims to be an extension of the results on the fixture design issues previously reported in [14].

3. Fixture Model

The fundamental performance of a fixture is characterized by the kinematic constraints imposed on the workpiece being held by the fixture. The kinematic conditions are well understood [3, 4, 5, 7, 12]. For a fixture of n locators \((i = 1, 2, \ldots, n)\), the fixture can be represented by:

\[
\delta y = G^T \delta q
\]  

(1)
where \( \delta y = [\delta y_1, \delta y_2, \ldots, \delta y_n]^T \) and \( \delta q = [\delta r, \delta \theta]^T \) define small perturbations in the locator positions and the location of the workpiece respectively. The fixture design is defined by the locator matrix \( G = [h_1, h_2, \ldots, h_n] \) where \( h_i^T = \{q_i^T (r_i \times n_i)^T \} \) and \( n_i \) and \( r_i \) denote the surface normal and position at the \( i \)th contact point on the workpiece surface. The problem of fixture design requires the synthesis of a fixturing scheme to meet a given set of performance requirements.

4. Quality Performance Criteria for a Fixture

4.1. Accurate Localization

An essential aspect of fixture quality is to position with precision the workpiece into the fixturing system. In general, the workpiece positional errors are due to the geometric variability of the part and the locators set-up errors. This paper will focus only on the workpiece positional errors due to the locator positioning errors.

As an extension of the fixture model equation (eq.1), the locator positioning errors \( \delta y \) can be related with the workpiece localization error \( \delta q \) as follows:

\[
\| \delta q \| = \delta q^T (G G^T) \delta q
\]

Clearly, for given source errors the workpiece positional accuracy depends only on the locator locations being independent from the clamping system, the Fisher information matrix \( M = G G^T \) characterizing completely the system errors. It has been shown [12] that a suitable criterion to achieve high localization accuracy is to maximize the determinant of the information matrix (D-optimality), i.e., \( \max(\text{det} M) \).

4.2. Minimal Locator Contact Forces

Another objective in planning a fixture layout might be to minimize all support forces at the locator contact regions throughout all the operations with complete kinematic restraint or force-closure. Locator contact forces in response to the clamping action are given as:

\[
\alpha_c = -G^T (G G^T)^{-1} h_i \lambda_c = -G^T M^{-1} h_i \lambda_c
\]

Normalizing these forces with respect to the clamping intensity we obtain:

\[
p_c = \alpha_c / \lambda_c = -G^T M^{-1} h_i, \text{ where } p = \{p_{ci} \}_{i=1}^n
\]

\[
p_{ci} = -h_i^T M^{-1} h_i, (i = 1, 2, \ldots, n)
\]

The force-closure condition requires these forces to be always positive for each locator \( i \) of a set of \( n \) locators:

\[
p_{ci} > 0, \text{ for } \lambda_c > 0
\]

Computing the norm of the locator contact forces:

\[
\| p_c \|^2 = \sum p_{ci}^2 = h_i^T M^{-1} h_i
\]

leads to an appropriate design objective, i.e., \( \min(\| p_c \|) \).

Note that this objective indicates both locator and clamp positions to be determinant in the optimization process.

4.3. Balanced Locator Contact Forces

Another significant issue in designing a fixture is that the total force acting on the workpiece have to be distributed as uniformly as possible among the locator contact regions. If \( \overline{p} \) represents the mean reactive force in response to the clamp action, then we define the dispersion of the locator contact forces as:

\[
d = \frac{1}{n} \sum_{i=1}^{n} (p_{ci} - \overline{p})^2 \text{ where } \overline{p} = \frac{1}{n} \sum_{i=1}^{n} p_{ci}
\]

Therefore, minimizing the defined dispersion represents an objective for a balanced force-closed fixture: \( \min(d) \).

5. Optimal Fixture Design with Interchange Algorithms

As mentioned earlier, by generating on the exterior surface of the workpiece to be fixtured a set of discrete locations defined as position and orientation, we create a potential collection for the fixture elements. For example, using the information contained in the part CAD model, a discrete vector collection (unitary, normal vectors) can be generated as uniformly as possible on those surfaces accessible to the fixture components (fig.1).

![Figure 1: Part CAD model and global collection of candidate locations for the fixture elements.](image)

The fixture design layout will select from this collection optimal candidates for locators and clamps with respect to the performance objectives and to the kinematic closure condition. Dealing with a large number of candidate locations the task of selecting an appropriate set of fixels is very complex.

As already introduced in [12, 14] an effective method for finding the desired fixture with regard to one of the previous quality objectives is the optimal pursuit method with an interchange algorithm. Due to its own limitations and to the fact that the objectives are functions with many extremes, the exchange procedure may not end up to a unique optimized fixture configuration, but to several improved designs depending on the initial layout. Therefore the solution offered by the multiple interchange with random initialization algorithm is overwhelming favorable, fact that recommends this procedure over the single interchange algorithms.

The algorithm can be described as a sequence of three phases:
**Phase 1: Random generation of initial sets of locators.**
The starting layout is generated by a random selection of distinct sets, each consisting from 6 locators out of the list of N candidate locations. If the clamp is pre-determined, a valid selection is obtained through a simultaneous check for all kinematic constraints. A big initial set of proposed locators is preferred, giving the opportunity of finding a convergent optimal solution. However from the efficiency point of view the designer has to balance the algorithm between the accuracy of the final solution and the computation time.

**Phase 2: Improvement by interchange.**
The interchange algorithm’s goal is to pursue for an improvement of the initial sets of locators with respect to one of the objectives. Basically, this is done iteratively by exchanging one by one the proposed locators with candidate locations from the global collection. It is also essential to consider the form-closure restraint during the exchange procedure. The process will continue as long as an improvement of the objective function is registered.

Studying the effect of interchange on the proposed quality measures leads us to some efficient algebraic properties. For example, an interchange between a current locator \( j \) \((j = 1,2,\ldots,N-6)\) and a candidate location \( k \) \((k = 1,2,\ldots,N-6)\) yields changes in the optimized function such that:

\[
\det M_{(j,k)} = p_{jk}^2 \left( \det M \right)
\]

\[
\|p_i\|^2 = \|p_i\|^2 + \frac{p_{ij}}{p_{jk}} \left[ p_{ij}(1 + p_{jk}) - 2p_{ik} p_{jk} \right] \|p_i\|^2 + \Delta p_i
\]

where 
\[
p_{ij} = -h_i M^{-1} h_j; \quad p_{jk} = -h_k M^{-1} h_i;
\]

\[
p_{jk} = h_j M^{-1} h_k; \quad p_{kk} = h_k M^{-1} h_k;
\]

Thus, at each interchange the pair is selected such that the significant term that controls the function evolution is improving, e.g. \( \max p_{jk}^2 \) and \( \min \Delta p_i \), easing the iterative process.

**Phase 3: Selecting the optimal solution.**
Applying the interchange algorithm for each initial set of locators we will end up with several distinct solutions on the configuration scheme of the fixture, the best fixture design corresponds evidently to the maximum improvement of the objective function. It should be emphasized that this algorithm can be used sequentially for different objective functions. Depending on the objective pursued the best solution can be evident (for a single objective) or might need the designer’s final decision (for multiple objectives).

6. **Fixture Locator Optimization**

In many applications the clamp is already fixed given some practical considerations. Then with the clamp pre-defined, the best fixture with respect to a certain performance criterion is constructed by selecting a suitable set of locators such that a significant improvement of the objective-function is registered.

Using the random interchange algorithm we can analyze the impact of the optimization process on the fixture characteristics, as well as we can select the best optimized fixture solution for a specific criteria. In analyzing the effect of random interchange algorithm on several parts, there can be made the following statistical and empirical observations.

6.1. **Single-objective Optimization**

1. **Accurate localization objective**

Optimization for a precise localization \( \max(\det M) \) objective would usually make significant improvement in the fixture quality. During the optimization process while the determinant is increasing, the general tendency of the norm or dispersion of the contact forces, with few exceptions, is a decreasing one (fig. 2). This indicates that benefit changes are recording for all objectives, seldom existing a conflict between them.

![Figure 2: Random interchange for \( \max(\det M) \) objective.](image)

Furthermore, the best fixture solutions present locators spread away as far as possible from each other, gathering very close to the object boundaries (fig. 3). Starting with a large collection of initial sets of locators, the interchange design process converges to a few final solutions. The local optimal solutions usually cluster around the best solution (fig. 3).

![Figure 3: Best fixture solution; collection of optimized solutions cluster together around the best solution.](image)

2. **Minimal locator contact forces objective**

Analyzing the impact of the optimization process for minimizing the locator contact norm forces \( \min(\|p_i\|) \).
some undesired changes are revealed. The determinant $\det(M)$ would often deteriorate significantly, affecting the localization quality of the part. Furthermore, we observed that one or more locators of the resulting fixture might have reactive forces close to zero, being almost inactive. To overcome this deficiency large acting forces must be imposed for the clamp, leading to excessive loads on workpiece and fixture. On the other hand, we noticed that the dispersion of the contact forces would generally decrease, being benefic for an equilibrated fixture design. This emphasis that there is no conflict between the locator norm and dispersion objectives (fig. 4).

![Figure 4: Random interchange for $\min(\|p_c\|)$ objective.](image)

The best-optimized fixture configuration can efficiently be constructed using the random interchange algorithm. Applying this procedure on a large initial set of locators, we end up with a collection of optimized configurations of different aspects, presenting locators spread all over the part surface (fig. 5). However, there exist a strong preference in the fixtures arrangement, one or more locators gathering on the opposite side of the part relative to the given clamp location.

![Figure 5: Best fixture solution; collection of optimized solutions obtained with random interchange.](image)

3. Balanced locator contact forces objective

Trying to distribute more uniformly the reaction forces to the clamping action between the locators, i.e. $\min(d)$, we notice a better behavior of the fixture characteristics compared with the previous objective. Eventhough the evolution tendency of the determinant can not be predicted, the fluctuations are relatively small (fig. 6). Also, the minimum value of the forces is pulled up from zero, implying that all locators are active and as a consequence the clamp force may be relaxed a lot. Therefore, we recommend strongly as a second objective in fixture design the minimization of the force dispersion.

![Figure 6: Random interchange for $\min(d)$ objective.](image)

![Figure 7: Best fixture solution; collection of optimized solutions obtained with random interchange.](image)

6.2. Multiple-objective Optimization

1. Multi-objective trade-offs

In some applications both localization quality and a minimum force dispersion are important. In this case we may have to use a 2-step algorithm: first $\max(\det M)$ and secondly $\min(d)$. The proposed order is a consequence of the above observations. First, maximizing the determinant will automatically decrease the dispersion, fact benefic; proceeding next, a decreasing in dispersion leads in a decreasing in determinant value. Therefore, during the second phase of the algorithm tradeoffs between the two objectives occur, fact expressed also through the Pareto-line plot (fig. 8). Thus, the designer decision is determinant in finalizing the fixture scheme.

![Figure 8: Behavior during a 2-step random interchange algorithm for a collection of locator sets.](image)
2. Designer decision in finalizing the fixture

During the second phase of the algorithm a fairly significant decrease in the determinant value is registered, so few solutions will be acceptable for the multi-objective problem. In order to overcome these problems, an active designer control during min($d$) interchange procedure is recommended. Essentially, the modifications consist in controlling the exchange procedure, such that the determinant of the improved locators must be permanently greater than a certain bound, simultaneously with the check for the form-closure condition. Even considering a tight bound for the determinant, more solutions are acceptable for the design than in the uncontrolled min($d$) optimization case (fig. 9).

As an example, the behavior of a single set of locators is studied during the interchange process of a 2-step algorithm controlled for two different bounds of the determinant value, emphasizing the fact that in the trade-off zone the designer decision is decisive in finalizing the fixture configuration (fig. 10).

Figure 9: Second phase of a 2-step random interchange algorithm: uncontrolled min($d$); controlled min($d$).

Figure 10: General behavior during a 2-step algorithm applied on a single set of locators.

7. Fixture Optimal Clamping

This section deals with a more complicated problem: to search simultaneously for the optimal clamp and locators in order to achieve a required fixture quality. Varying the clamp, it is obvious that the number of combinations for possible clamp-locators candidates is increasing very much. It will be shown that this problem is manageable for the precise localization objective. For the other objectives we will have to restrain the search of the optimal clamp inside of a small set of proposed locations, such that the optimization procedure could be handled.

7.1. Feasible Clamping Regions

Applying an automated procedure based on the derived conditions (eq.5) for given sets of locators we may identify the entire feasible regions of clamp locations that would satisfy the form-closure requirement. Fig. 11 illustrates the feasible clamping regions for different sets of locators. It can be noticed that the feasible clamps, if there exist, are gathered in compact regions over the part surface. Their number, distribution and aspect are completely determined of the locator scheme configuration. It is obvious that the locators that correspond to no clamp solution are excluded to form a feasible fixture.

Figure 11: Feasible clamping regions.

7.2. Optimal Clamping for Precise Localization

For precise applications the main concern in fixture design is the precision of workpiece localization. Thus, the primary objective is max(det $M$) while secondary objectives may include $\min \| p_c \|$ or $\min(d)$.

As shown previously, a highly accurate localization of a part is dependent only on locator positions. Without considering clamps (i.e., the form-closure constraint), the interchange algorithm for max(det $M$) would give us the best solution for optimal localization. Once the locators are determined, we may identify the entire feasible clamping regions for a form-closed fixture. These will represent the optimal clamping regions that assure the highest accurate localization of the part. If such regions exist, we have the opportunity to choose a clamp for a desired second objective completing the design (fig. 12). When no feasible clamp regions exist, this hierarchical design process would fail, and we have to compromise between the precise localization and feasible clamping.

Figure 12: Symmetrical solutions for best locators and the corresponding optimal clamp regions for max(det $M$).
7.3. Optimal Clamp from a Set of Clamps

In some applications the clamps have certain preferred locations, therefore the need to choose the best clamp from a proposed collection might be raised.

For example, let's consider that a collection of preferred clamps is given, and an optimal fixture design with respect to the highly precise localization objective is needed. It is obvious that applying a random interchange procedure successively for each clamp, we find optimal fixture configurations for each specified clamp. Comparing the determinant values offered by these fixture schemes (fig. 13), we end up by selecting an optimal clamp and its corresponding locators, constructing the best-improved fixture design (fig. 14).

Furthermore, by extension, the selection of the optimal clamp from a set of proposed locations with regard to the multi-objective design problem can be considered. It will consist mainly in applying the random 2-step interchange algorithm consecutively for each proposed clamp.

By collecting the results after applying this procedure for all the clamps, we can compare their different behavior, and select the most appropriate one. It is obvious that an optimal clamp allows only small fluctuations of the determinant while the force dispersion is decreasing significantly (fig. 15). As an example, fig. 16 illustrates the final fixture design consisting of the best clamp selected from a proposed collection with respect to the multi-objective problem, and the corresponding optimal locators.

8. Conclusions

This article focuses on optimal design of fixture layout for 3D workpieces with an optimal random interchange algorithm. The quality objectives considered include accurate workpiece localization, minimal and balanced contact forces. A thorough analysis is performed on the optimization process with respect to the performance objectives. Several main directions for the 3D-fixture synthesis, such that single-criteria optimal design with clamp pre-defined and multi-criteria optimal design with hierarchical approach and combined-objective approach, have been solved. Examples are used to illustrate some empirical observations with respect to the design approach and its effectiveness.

The work described here is just a tip of the iceberg. The inter-relationship between the locators and the clamps has a determinant role on the fixture quality measures. A more coherent and complete approach to study the influence of the clamp and on an unlimited search of the optimal clamp position must be subject of further studies.

References

3. B. Mishra, J. T. Schwartz, and M. Sharir, "On the existence and synthesis of multifinger positive grips"