Reactive Behaviours of Mobile Manipulators
Based on the DVZ Approach

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Abstract
This paper addresses the problem of reactive behaviours of mobile manipulators evolving in dynamic and unknown environments. The Deformable Virtual Zone (DVZ) method is proposed to resolve the problem through the reflex action theory. The formulation of the DVZ principle is given and the extension to mobile manipulators is detailed. This approach allows to implement fast control laws and can be seen as an efficient low level algorithm for controlling motions to avoid unscheduled obstacles collisions. Simulation results are discussed, showing the effectiveness of the proposed method.

1 Introduction
Artificial reflex actions for mobile robots can be defined as the ability to react when unscheduled events occur, for instance when they move in unknown and dynamic environments. The problem of designing reflex-oriented artificial systems has been partially solved by using a behavioural approach, consisting in directly relating inputs (stimuli) to outputs (actions) through state machines and to make these elementary machines communicate [1][2][3][4]. The ways these machines are programmed are various: fuzzy logic, neural networks, deterministic state machines and so on. Another design consists in considering a sensor-based approach which feedbacks sensory information to the robot control loop [5][6]. The most famous method of this family is the potential method developed by O. Khatib, twenty years ago [7].
In the first case, the stimuli are translated into virtual external forces which are simply added to the force due to the goal, when there is a goal, and all these forces produce an action, moving the system and locally modifying the world. It is quite clear that the second approach is not so different: the information provided by exteroceptive sensors generate the same virtual external forces which contribute to the system evolution, after a comparison with the forces due to the programmed goal.
For the last nine years, we have been interested in the problem of reactive behaviours for collision avoidance in the domain of mobile robotics. We have investigated the control of land autonomous vehicles, underseas robots, manipulators and walking machines. Many experiments have been carried out on real robots (wheeled mobile robots, legged robots and an AUV) and on simulated ones. We have developed a collision avoidance method called DVZ (Deformable Virtual Zone) that allows to control the reactive behaviours of a mobile robot [8]. In this paper we propose an extension of this method to the case of mobile manipulators.
This paper is organized into five sections. In section 2 the DVZ principle approach in its general frame is described. The extension to mobile robotic manipulators is presented in section 3, and in section 4 simulation results are discussed. Conclusions are given in sect. 5.

2 The DVZ Principle
This paragraph describes the DVZ principle in which a rigid body (the robot), evolving in an unknown environment, is supposed to be surrounded by a Deformable Virtual Zone (DVZ), the geometry of which depends on the body generalized coordinates (state) and whose deformations are due to the interaction with the environment.

Definition 1 For a convex rigid body \( R \subset \mathbb{R}^3 \), we define the undeformed DVZ-set \( \Xi_h \) as any convex surface surrounding \( R \) and verifying a one-one correspondence with the boundary \( \partial R \) of \( R \).

Definition 2 The undeformed DVZ \( \Xi_h \) of a convex rigid body \( R \subset \mathbb{R}^3 \), is the one-one map in the set of convex surfaces, relating \( \partial R \) to \( \Xi_h \) and formally defined by: \( M \overset{\Xi_h}{\rightarrow} P_h \).

Following the definition, \( \Xi_h = \Xi_h (\partial R) \).
Definition 3 For a convex rigid body moving among obstacles, and for which is defined a DVZ-set $\Xi_0$, we define the deformed DVZ-set $\Xi$ as any convex surface surrounding $R$ and included in $\Xi_0$ and verifying a one-one correspondence with the boundary $\partial R$ of $R$.

Definition 4 The deformed DVZ $\Xi$ of a convex rigid body $R \subset \mathbb{R}^3$, is the one-one map in the set of convex surfaces, relating $\partial R$ to $\Xi$ and formally defined by: $M \to \Xi \to \Xi_0$.

Following the definition, $\Xi = \Xi(\partial R)$.

Definition 5 The deformation $\Delta$ is defined as the functional difference of $\Xi$ and $\Xi_0$:

$$\Delta = \Xi - \Xi_0$$ (1)

The deformation $\Delta$ is a one-one map that associates the vector $P - P_0$ to the point $M \in \partial R$. It can therefore be considered as a vector field defined on $\partial R$.

The undeformed DVZ depends on the state vector $\pi$ characterizing the motion capabilities of the body (its translational and rotational velocities for instance):

$$\Xi_0 = \beta(\pi)$$ (2)

Let $M$ be a point of the boundary of $R$, $M \in \partial R$. The deformation vector $\Delta(M)$ depends on $I$, intrusion of the environment into the body space at $M$ and on the undeformed DVZ at $M$:

$$\Delta(M) = \alpha(\Xi_0(M), I(M))$$ (3)

In the following we refer to (3) as:

$$\Delta = \alpha(\Xi_0, I)$$ (4)

implicitly implying the vectors image of $M$ through the correspondent map. Differentiating equation (4) yields:

$$\dot{\Delta} = \nabla_{\Xi_0}[\alpha] \nabla_\pi[\beta] \phi + \nabla_I[\alpha] \psi$$ (5)

where $\nabla_\xi$ is the derivation operator with respect to the variable $\xi$ and where $\phi = \nabla_\pi \pi = \pi$ and $\psi = \nabla_I I = \dot{I}$ are the two control vector of $\Delta$.

The evolution of $\Delta$ is driven by a two-fold input vector $u = \begin{bmatrix} \phi, \psi \end{bmatrix}^T$. The first control vector $\phi$, due to the controller, tends to minimise the deformation of the DVZ. The second one, $\psi$, is unknown and is induced by the environment itself (and could, at most, try to maximize these deformations). The complete evolution of the deformation is modeled by a differential equation of the type:

$$\dot{\Delta} = A \phi + B \psi$$ (6)

The DVZ control algorithm consists of choosing the desired evolution $\Delta_{des}$ of the deformation and applying the following lemma:

Lemma 1 (DVZ Principle) Given $\Delta_{des}$, the best control vector $\dot{\phi}$ in the sense of least-squares, that minimises function $\| \Delta_{des} - \dot{\Delta} \|^2$ is obtained by inverting equation (6):

$$\dot{\phi} = A^+ (\Delta_{des} - B \psi)$$ (7)

where $A^+$ is the pseudo-inverse of $A$.

A simple and efficient control law consists of choosing the desired deformation as proportional to the real deformation and its derivative:

$$\dot{\Delta}_{des} = -K_p \Delta - K_d \dot{\Delta}$$ (8)

where the two matrices $K_p$ and $K_d$ are respectively the proportional and derivative gain and are tuned in order to carry out the avoidance task.

2.1 Extension to Non-Rigid Bodies

We now extend the DVZ principle to the case of a non-rigid body $R$, i.e. a body presenting an internal configuration that can vary in time. Let $\eta$ be the internal state of $R$. The internal configuration of $R$ is completely described by $\eta$.

We define the undeformed DVZ $\Xi_0$ of $R$ by applying the general definition 2.

The undeformed DVZ of a non-rigid body depends both on the state vector $\pi$ characterizing the body motion capabilities and on the internal state $\eta$:

$$\Xi_0 = \beta(\pi, \eta)$$ (9)

Following the same conceptual development of the precedent paragraph, differentiating (4) yields:

$$\dot{\Delta} = \nabla_{\Xi_0}[\alpha] \nabla_\pi[\beta] \phi + \nabla_\eta[\alpha] \psi + \nabla_\Xi[\alpha] \nabla_\eta[\beta] \xi$$ (10)

where $\xi = \nabla_\eta \eta = \dot{\eta}$ is the evolution of the internal state.

The complete evolution of the deformation $\Delta$ for a non-rigid body is modeled by a differential equation of the type:

$$\dot{\Delta} = A \phi + B \psi + C$$ (11)

This result represents the extension of (5) to non-rigid bodies. In this case the complete evolution of the deformation vector $\Delta$ depends as well on the internal configuration. We point out that $\nabla_\eta[\beta] = \nabla_\phi[\beta] \nabla_\eta[\Theta]$ and $\nabla_\eta[\Theta] = \nabla_\eta[\Theta](\eta, \dot{\eta})$, being $\Theta = \Theta(\eta, \dot{\eta})$: the relation between $\Delta$ and $\psi$ is non linear. In general $\dot{\eta}$ is a subset of $\pi$. 681
3 Mobile Manipulators

In this section we consider one link of the mobile manipulator and we aim to apply the DVZ principle. Firstly we define a meaningful shape for the link’s DVZ and we express the deformation as a function of the mobile manipulator configuration and kinematic. Secondly we apply the extension of the DVZ control algorithm to generate the reflex command and let the reactive behaviour emerge.

A mobile manipulator can be characterized as a non-rigid body, presenting internal configurations defined by \( \eta = (\theta, \gamma) \) where \( \theta \) represents the joint angles of the manipulator and \( \gamma \) the absolute orientation of the mobile base. We expect to find the same differential equation (11) to describe the complete evolution of \( \Delta \).

3.1 Notation

We define the notation as follows: \( \Sigma_L, \Sigma_B, \Sigma_S, \Sigma_0 \) respectively the link, mobile base, sensors and absolute frames; \( ^2T_1 = \begin{bmatrix} ^2R_1 & ^2O_1 \\ 0 & 1 \end{bmatrix} \), \( ^2R_1 \) and \( ^2O_1 \) respectively the homogeneous transformation matrice, rotation matrice and relative position from \( \Sigma_1 \) to \( \Sigma_2 \); \( ^0t_{1,2} = \begin{bmatrix} ^0v_{1,2} & ^0\omega_{1,2} \end{bmatrix}^T \) the six-dimensional generalized velocity vector, referred to as twist, of \( \Sigma_1 \) relative to \( \Sigma_2 \), expressed in \( \Sigma_0 \); \( \mathbf{P} \) a vector, \( P = ||\mathbf{P}|| \) its norm, \( \mathbf{\hat{P}} = \frac{\mathbf{P}}{P} \) its direction. It follows \( \mathbf{P} = \mathbf{P} \cdot \mathbf{\hat{P}} \).

In the following all variables are expressed with respect to \( \Sigma_L \), link frame if not otherwise mentioned. \( \Sigma_L \) for link \( i-t \) is defined as follows: axis \( x \equiv \text{symmetry axis of the link; origin} \ O_L \equiv \text{center of joint} \ i+1 \).

3.2 DVZ Definition

A natural choice to define the robotic arm’s DVZ map \( \Xi_h \) of a mobile manipulator, is to associate the arm with a cylinder-like surface surrounding each link \( R \). By doing so, a safe protective zone for each link (therefore for the whole arm) towards all directions is guaranteed. We define the undeformed DVZ set as:

\[
\Xi_h = \left\{ \mathbf{P}^h : \forall \mathbf{M} \in \partial R, \ P_x^h = M_x, \ \mathbf{\hat{P}}_y^h = M_{yz}, \ P_y^h = \left( \sqrt{\mathbf{M}_{yz} \cdot \mathbf{A} \cdot \mathbf{M}_{yz}} \right)^{-1} + d_{min} \right\} \tag{12}
\]

where

\[
\mathbf{A} = \mathbf{V} \cdot \Lambda \cdot \mathbf{V}^T, \quad \Lambda = \begin{bmatrix} \frac{1}{c_y^2} & 0 \\ 0 & \frac{1}{c_y} \end{bmatrix} \tag{13}
\]

\[
\mathbf{V} = \frac{1}{||\mathbf{v}_M||} \begin{bmatrix} \mathbf{v}_M |_y & -\mathbf{v}_M |_z \\ \mathbf{v}_M |_z & \mathbf{v}_M |_y \end{bmatrix} \tag{14}
\]

\[
c_y = (1-r) \cdot ||\mathbf{v}_M|| \cdot c_z = (1-f) \cdot c_y \tag{15}
\]

Following the definition, \( \Xi_h \) describes a generalized cylinder whose section is a variable ellipse either in size and orientation, refer to fig.1 and 2.

![Figure 1: 3-D representation of link DVZ](image1)

![Figure 2: link DVZ cross sections](image2)

Vector \( \mathbf{v}_M |_{yz} \) is the \( yz \) component of \( \mathbf{v}_M \), velocity of point \( M \). Only the \( yz \) component of the velocity vector affects the DVZ since the \( x \) component represents a translation towards the link’s axis and it will be duty of the DVZ of the successive or precedent link in the robotic arm chain (the end-effector or mobile base if a terminal link) to prevent collisions on that direction.

Parameter \( d_{min} \) defines a minimum safety zone around the link, inside which no intrusion is allowed. Normalized parameter \( f \) is the fineness of the DVZ, or width/length ratio. Normalized parameter \( r \) is the risk: a high value of \( r \) (\( \geq 1 \)) means small DVZ, weakly dependent on state vector \( \pi \), whereas a low value of \( r \) (\( \leq 0 \)) means large DVZ, strongly dependent on state vector \( \tau \). The three parameters \( (r, f, d_{min}) \) are referred to as the intrinsic parameters of the DVZ.
The velocity $\mathbf{v}_M$ is calculated by applying the direct kinematic to the mobile manipulator. We define the matrix operator $S(\mathbf{a})$ as:

$$S(\mathbf{a}) = \begin{bmatrix} a_x & a_y & a_z \\ -a_z & 0 & -a_y \\ a_z & 0 & -a_x \end{bmatrix}$$

so that $\mathbf{b} \times \mathbf{a} = S(\mathbf{a}) \cdot \mathbf{b}$. Using the above expression, $\mathbf{v}_M$ is given as:

$$\mathbf{v}_M = [L \mathbf{R}_B \quad S (\mathbf{M}) \cdot L \mathbf{R}_B] \cdot B \mathbf{t}_{L,0}$$

$L \mathbf{R}_B$ represents the rotation matrix of $\mathbf{M}_B$ with respect to $\mathbf{S}_L$ and is a function of the manipulator configuration $\theta$. The six-dimensional vector $B \mathbf{t}_{L,0}$ represents the twist of $\Sigma_L$ with respect to $\Sigma_0$ expressed in $\Sigma_B$ and is given as:

$$B \mathbf{t}_{L,0} = \mathbf{J} \cdot \dot{\theta} + \begin{bmatrix} \mathbf{I} \\ S (B \mathbf{O}_L) \end{bmatrix} \cdot B \mathbf{t}_{B,0}$$

where $\mathbf{J}$ is the Jacobian matrix for the link.

The mobile base twist vector $B \mathbf{t}_{B,0}$ is given as $B \mathbf{t}_{B,0} = \chi (v_B, \dot{\gamma}_B, \gamma)$, where $v_B$ and $\dot{\gamma}_B$ are the controlled velocities of the mobile base. Function $\chi(\cdot)$ depends on the motion capabilities of mobile base, i.e., on its degrees of freedom, and can account for non-holonomic constraints. For instance, considering the case of an holonomic mobile base moving in the $xy$ plane, $v_B$ represents the absolute value of the linear velocity towards the front and the side of the base $(v_B)_x$ and $(v_B)_y$ whilst $\dot{\gamma}_B$ the rotation velocity about the vertical axis $z$.

This result, combined with the above considerations, clearly underlines the dependence of the undeformed DVZ $\Sigma_h$ on the mobile manipulator motion capabilities, by means of the state vector defined as $\pi = [\dot{\theta}, v_B, \dot{\gamma}_B]^T$, and on the mobile manipulator internal configuration $\eta$, that is $\Sigma_h = \Sigma_h (\pi, \eta)$.

The description of the environment surrounding the manipulator is given as $I$, proximity measurements by means of any sensorial equipment fitted on the mobile manipulator. Without losing in generality, we consider $I$ referred to the sensor frame $\Sigma_s$. The intrusion of the environment inside the link space is given by the deformed DVZ $\Sigma$, defined as:

$$\Sigma = \{ \mathbf{P} : \mathbf{P} \text{ inside } \Sigma_h, \mathbf{P} = L \mathbf{R}_S(\theta) \cdot I + L \mathbf{O}_S \}$$

Referring to fig.2, to each point $\mathbf{P} \in \Sigma$ corresponds, through $\Sigma$, one point $\mathbf{M}$ on the boundary of the link such that:

$$\forall \mathbf{P} \in \Sigma \Rightarrow \mathbf{M} \in \partial R, \quad M_x = P_x, M_{yz} = \hat{P}_{yz}$$

Applying definition 5, the deformation vector field $\Delta$ is given as follows:

$$\Delta = \left\{ \Delta : \forall \mathbf{M} \in \partial R, \quad M_x = 0, \quad \Delta_{yz} = \hat{P}_{yz} \cdot (P_{yz} - P_{yz}^0) \right\}$$

It is easy to verify that $\Delta = \Delta (\Sigma_h, I)$, as expected by the general DVZ theory for non-rigid bodies.

### 3.3 Generation of Reflex Command

We develop the fundamental equation to generate the reflex command by applying the DVZ control algorithm for non-rigid bodies. The goal of the control law is to minimise the norm of the deformation vector $\Delta$, expressible from (21) as:

$$\Delta = P_{yz} - \frac{1}{\sqrt{\hat{P}_{yz}^T \cdot A \cdot \hat{P}_{yz} + d_{min}}}$$

where we have substituted (12). Differentiating (22) yields:

$$\Delta = \nabla_{\pi} [\Delta \hat{\pi} + \nabla_{\tau} [\Delta \hat{\tau} + \nabla_{\eta} [\Delta \hat{\eta}]]$$

where $\hat{A} = \nabla_{\pi} [\Delta \hat{\pi}], \hat{B} = \nabla_{\tau} [\Delta] \quad \text{and} \quad \hat{C} = \nabla_{\eta} [\Delta \hat{\eta}]$.

We assume that distance measurements $(\rho)$ towards $m$ directions $(\mathbf{u})$ of the three-dimensional space surrounding the mobile manipulator are available. We define $I = [\rho_1, \rho_2, \ldots, \rho_m]^T$, where $\rho_m$ is the measurement towards $\mathbf{u}_m$ and $\mathbf{u}_n$ is considered fixed in $\Sigma_S$. It follows that $\psi = I = [\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_m]^T$. As already mentioned we have no control capabilities on $\rho$ (i.e. $I$), being induced by the environment.

![Figure 3: Sampled Deformation of DVZ](image-url)

Therefore (24) represents a matrice equation corresponding to the system of equations below:

$$\begin{align*}
\Delta_1 &= A_1 \cdot \phi + B_1 \cdot \psi_1 + C_1 \\
\Delta_2 &= A_2 \cdot \phi + B_2 \cdot \psi_2 + C_2 \\
&\vdots \\
\Delta_m &= A_m \cdot \phi + B_m \cdot \psi_m + C_m
\end{align*}$$

(25)
Since we have knowledge of the environment towards \( m \) fixed directions, the \( i-th \) equation of system (25) represents the DVZ principle applied to point \( M_i \) of the link, defined as in (20) for each of the \( m \) point \( P_i \) belonging to \( \Xi \) (refer to fig.3).

Considering each link constituting the manipulator, an equation on the form of (24) is calculated leading to the following system:

\[
\begin{align*}
\hat{\Delta}_{\text{link}1} &= A_{\text{link}1} \cdot \phi + B_{\text{link}1} \cdot \psi + C_{\text{link}1} \\
\hat{\Delta}_{\text{link}2} &= A_{\text{link}2} \cdot \phi + B_{\text{link}2} \cdot \psi + C_{\text{link}2} \\
&\vdots \\
\hat{\Delta}_{\text{link}n} &= A_{\text{link}n} \cdot \phi + B_{\text{link}n} \cdot \psi + C_{\text{link}n}
\end{align*}
\]

(26)

that we can compact in a matrix form as:

\[
\hat{\Delta}_{\text{arm}} = A_{\text{arm}} \cdot \phi + B_{\text{arm}} \cdot \psi + C_{\text{arm}}
\]

(27)

This result represents the complete evolution of the deformation for the whole manipulator. The DVZ control algorithm is next applied choosing the following control law:

\[
\dot{\Delta}_{\text{arm}}^{\text{des}} = -K_p \cdot \Delta_{\text{arm}} - K_d \cdot \dot{\Delta}_{\text{arm}}
\]

(28)

The collision avoidance control vector is given as:

\[
\dot{\phi} = A_{\text{arm}}^+ \cdot \left( \Delta_{\text{arm}}^{\text{des}} - B_{\text{arm}} \cdot \psi - C_{\text{arm}} \right)
\]

(29)

and is then applied to the mobile manipulator.

### 4 Simulation Results

In this section we present some simulation results. A graphic simulator has been developed and implemented to test the effectiveness of the proposed approach for collision avoidance from either a computational and behavioural point of view. The simulator constitutes an interface that allows us to interactively change the position of the obstacles, the intrinsic parameters of the DVZ, the mobile manipulator configuration and initial state. It permits as well to visualize the three-dimensional motion of the mobile manipulator and the geometry of the DVZ. We discuss the behaviour of a simulated mobile manipulator looking at some typical situations.

In the following experiments, we consider an holonomic mobile base, moving in a two-dimensional space, together with a two-link planar manipulator. The mobile manipulator moves straight forward from initial point \( A \) to final point \( B \) at constant speed, keeping the arm still in its initial position. If an unscheduled obstacle is found on the path, the collision avoidance algorithm takes over, the reactive behaviour emerges and the configuration of the mobile manipulator changes in consequence. The mobile robot is equipped with 48 simulated ultrasonic sensors, 24 fitted on the mobile base and 24 on the terminal link, pointing to the surrounding two-dimensional space and measuring the distance towards a fixed direction. Manipulator interferences on sensor measurements are not taken in account. Ultrasonic sensors have already been used in experiments carried out on real mobile robots for reflexive collision avoidance using the DVZ algorithm[8]. The particular choice of sensorial equipment does not limit the application of the method. In fact, for mobile manipulators other systems of measurement could be used such as a stereoscopic-vision system, which measures the distance of the obstacles towards all directions within the scope of the cameras[9].

This is the actual ongoing research project of our laboratory.

Comparison between different situations are carried out. The figures show a mosaic of the mobile manipulator positions at every sample time. An intensification of darkness in the mobile base trail represents a decrease of speed.

![Figure 4: 2dof manipulator, 1dof base](image)

In fig.4 the mobile base is allowed to move backward and forward only and the manipulator is two-link planar, where only the second link is shown for clearness. The control vector is therefore the three-fold vector \( \pi = [\theta_1, \theta_2, v_p]^T \).

The same simulation with different values of risk are compared. From 4(a) to 4(c) an increasing value of risk is chosen, consequently the size of the DVZ is smaller and the reflexive behavior emerges later (closer to the obstacle). In 4(d) two obstacles are considered, simulating a narrow passage as in entering through a door and in 4(e) a wall-like surface is introduced.

In fig.5 comparison between particular situations are presented. Fig.5(a) points out the need to attentively choose the sensing equipment, in order to avoid unreliable situation leading to undesired collisions. The big obstacle on the right shadows the mobile base ultrasonic sensors, preventing the manipulator’s second link’s DVZ to sense the intrusion. Equipping the robotic manipulator with (ultrasonic) sensors fitted on the end-effector, for example, avoids this
situation leading to the correct behaviour 5(b). In fig.5(c) the base is allowed to rotate about the vertical axis $z$ and the non-holonomic constraint is introduced. A one-link manipulator is considered. The results show that the reflexive behaviour due to intrusion in the arm’s DVZ would cause a rotation of the base rather than the link, as reasonable, leading to this sort of attraction towards the obstacle that represents a dangerous situation. Thus, the mobile base DVZ is introduced and the correct behaviour emerges again, fig.5(d).

Finally, fig.6 shows a one link manipulator with spherical joint (shoulder joint of an antropomorph robotic manipulator) avoiding an obstacle. The same simulation with different values of risk are compared confirming the good results and generality of the algorithm.

## 5 Conclusion

This paper has presented an algorithm for implementing collision avoidance reactive behaviours on mobile manipulators. The method is based on an extension of the DVZ principle and it has been developed by the authors. The approach consists of defining a Deformable Virtual Zone surrounding the mobile manipulator as a function of the motion capabilities of the robot. The DVZ is deformed by the interactions with the environment. The complete evolution equation of the deformation is calculated and the best control law is obtained. Simulation results show the effectiveness of the method. We plan to carry out experiments on real mobile manipulators using motion vision as a fundamental element in the perception of the environment in order to prove in practice the power of the proposed design. The proposed algorithm permits to pursue simultaneously both reactive behaviour, collision free interaction and short term target following in a sensor based environment. It must be underlined the DVZ principle cannot be implemented alone in the sense that it does not implement a planning procedure. These properties make the reflex algorithm amenable to control hierarchies and makes it possible to use it for preservation and as a building block in efficient path planning algorithms.

## References


