Whole-Arm Impedance of a Serial-Chain Manipulator

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Abstract

In this paper, we propose a mechanical impedance for the whole arm of a serial-chain manipulator. The proposed impedance has a geometrically natural property owing to use of a spatial curve which describes a virtual reference of the impedance. The proposed impedance is useful for geometric task planning in whole-arm manipulation. Illustrative examples are shown for understanding of the impedance.

1 Introduction

Observing the object manipulation by humans, we notice that the applied forces are widely distributed over the object. This rich force distribution enables humans to achieve dexterous manipulation. In order to generate a variety of forces at one time by a robotic manipulator, we need the manipulation to utilize not only the tip but also the whole body. This type of manipulation is known as Whole-Arm Manipulation (WAM, for short). The goal of this research is to build up a fundamental control theory of WAM.

The concept of WAM was proposed by Salisbury in 1987 [10]. He and his colleagues also designed and developed hardware for WAM [11, 13].

The WAM concept has been accepted in the research on grasping by a robotic hand. Enveloping grasp is named by Trinkle et al. as the one using the surface of the hand, not just the finger tips. Power grasp is a kind of enveloping grasp whose mechanism can resist passively against external forces without relying on joint torques [2, 6, 14]. Enveloping grasp and power grasp have been studied actively even now [5, 12].

On the other hand, the WAM concept is also growing up in the research on manipulator control as the shape control scheme. In the shape control scheme, we control the shape of a manipulator directly by bringing all the joint position onto the prescribed desired spatial curve. However, in this scheme, the interaction with the manipulated object has not been considered yet. This means that the analysis of the force distribution of a manipulator in shape control is an open problem. (For more detail about the history, see [1, 8] and references there in.)

In this paper, based on the shape control scheme, we propose the mechanical impedance for the whole arm. Mechanical impedance is the fundamental concept for expressing the dynamic stiffness of a mechanical system. In robotics, the impedance control for the tip of a manipulator was proposed by Hogan in 1985 [4]. We try to extend the concept of tip impedance naturally to whole-arm impedance. The extension to whole-arm impedance in this paper greatly owes to the research on geometric tip-impedance by Fasse and Broenink [3]. Based on the whole-arm impedance proposed in this paper, we can expect to achieve WAM by simple shape control, e.g. whole-arm grasping.

In Section 2, we show novel kinematics for the whole arm of a serial-chain manipulator with 2DOF joints. Based on the kinematics, we discuss the whole-arm impedance with a compliance and damping in Section 3. We also show the corresponding joint torques. Illustrative examples are given in Section 4. We summarize the results of this paper in Section 5.

2 Whole-Arm Kinematics

2.1 Shape and Force Distribution

Euclidean space is denoted by $E^3$. Let $V^3, U^3 (\subset V^3)$ and $\Sigma^3$ be a space of geometric vectors, unit-length vectors, right-handed frames respectively. We also define $C := \{ -\pi, \pi \}$. The manipulator considered in this paper is a serial chain of $(n+1)$ rigid links con-
connected with \( n \) 2DOF rotational joints. The one end of the manipulator is fixed to the base which does not move relative to the ground. Starting from zero for links, one for joints, we number links and joints of the manipulator from the base side to the tip.

We attach a frame, \( F_i \in \Sigma^3 \), to each link so as to align its x-axis with the direction of the link length. Let \( p_i \in E^3 \) be the rotational center of the \((i + 1)\)-th joint. We call \( p_i \) and \( F_i \) the link point and the link frame respectively. Let \( F_0 \) and \( p_0 \) be the reference frame and its origin. Let \( p_i \in \mathbb{R}^3 \) and \( \Phi_i \in SO(3) \) denote the link position and orientation relative to the reference. Then, the kinematic equations of the manipulator can be described by the following recursive form:

\[
\Phi_i = \Phi_{i-1} R_{J,i} ,
\]

\[
p_i = p_{i-1} + l_i \Phi_i e_x ,
\]

where \( R_{J,i} \in SO(3) \) denotes the rotational action of the \( i \)-th joint, \( l_i \) is a positive real constant corresponding to the link length, and \( e_x \) is the x-directional unit-length vector. For more details about the kinematics of a manipulator with 2DOF joints, see [7].

Suppose that external force \( \Delta f_i \in \mathbb{R}^3 \) is applied at point \( p_{c,i} \) in the \( i \)-th link. Let \( p_{c,i} \) be the position vector of \( p_{c,i} \) relative to the reference. Then, the force and moment of the \( i \)-th link, \( f_i \in \mathbb{R}^3 \) and \( m_i \in \mathbb{R}^3 \) can be described by

\[
f_i = f_{i+1} + \Delta f_i ,
\]

\[
m_i = m_{i+1} + (p_i - p_{i-1}) \times f_{i+1} + (p_{c,i} - p_{i-1}) \times \Delta f_i .
\]

Let \( \tau_i \in \mathbb{R}^2 \) be the joint torque vector of the \( i \)-th link. The joint torque has the following relationship to the link moment:

\[
\tau_i = A_i^T m_i ,
\]

where \( A_i \in \mathbb{R}^{3 \times 2} \) is the matrix whose columns are the two axis-vectors of the \( i \)-th joint.

Define the shape of a manipulator \( p \in (\mathbb{R}^3)^n \) by

\[
p := [p_1^T \cdots p_i^T \cdots p_n^T]^T .
\]

Let \( \theta_i \in C^2 \) be the joint angle vector of the \( i \)-th joint, and define \( \theta \in (C^2)^n \) by

\[
\theta := [\theta_1^T \cdots \theta_i^T \cdots \theta_n^T]^T .
\]

We assume that the Jacobian matrix of \( p \) with respect to \( \theta , \frac{\partial p}{\partial \theta} \) is of full rank. We also assume that the contact of the \( i \)-th link occurs at \( p_i \). Define the joint torque vector \( \tau \in (\mathbb{R}^2)^n \) and the external force distribution \( \Delta f \in (\mathbb{R}^3)^n \) by

\[
\tau := [\tau_1^T \cdots \tau_i^T \cdots \tau_n^T]^T ,
\]

\[
\Delta f := [\Delta f_1^T \cdots \Delta f_i^T \cdots \Delta f_n^T]^T .
\]

From (3) and (4), we obtain the following relationship between \( \tau \) and \( \Delta f \):

\[
\tau = \frac{\partial p^T}{\partial \theta} \Delta f .
\]

### 2.2 Shape Space

Note that \( p \) is not free in \((\mathbb{R}^3)^n\), but restricted in the subset of \((\mathbb{R}^3)^n\), because of (2). Let \( Q \) be the subset which \( p \) belongs to, i.e., \( p \in Q \). We call \( Q \) the shape space.

The shape velocity at \( p \) is denoted by

\[
(p, v) \in T^* Q ,
\]

where \( v \) is the time derivative of \( p \), and \( T^* Q \) denotes the tangent space of \( Q \). We call \( T^* Q \) the shape-velocity space.

From (10) and the definition of \( v \), the work generated by applying the external force over a time interval \([t_1, t_2] \), \( W \), is expressed as

\[
W = \int_{t_1}^{t_2} \tau^T \dot{\theta} \, dt = \int_{t_1}^{t_2} \Delta f^T v \, dt .
\]

Therefore, the force corresponding to the shape velocity \((p, v)\) is denoted by

\[
(p, \Delta f) \in T^* Q .
\]

In other words, the force distribution, \( \Delta f \), is the dual value of \( v \). The space \( T^* Q \), the dual space of \( TQ \), is called the force-distribution space.

### 2.3 A Spatial Curve

A spatial curve can be expressed by a mapping of \( \mathbb{R} \) to a Euclidean space, \( E^3 \). Let \( c : \mathbb{R} \to E^3 ; \xi \mapsto c(\xi) \) be the mapping expressing a spatial curve, where \( \xi \in \mathbb{R} \) is the curve parameter. Assume that the curve \( ^\dagger \)

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\(^\dagger\)This assumption helps us to see the story of this paper well, although more general contact cases including multiple contacts on one link can be treated in the similar manner.
is sufficiently smooth, does not intersect itself, and any tangent vectors on the curve are normalized with respect to \( \xi \).

Define the shape specified by a curve, \( p_d \in (\mathbb{R}^3)^n \), by

\[
p_d(\xi) := \begin{bmatrix} c(\xi_1)^T & \cdots & c(\xi_i)^T & \cdots & c(\xi_n)^T \end{bmatrix}^T.
\]

where \( \xi := [\xi_1 \cdots \xi_n]^T \in \mathbb{R}^n \). The parameter \( \xi \) is called the curve-parameter vector. Note that the vector \( p_d \) is not free in \((\mathbb{R}^3)^n\) as well.

By using the shape by a curve, we can define shape error between a manipulator and curve, \( e \in (\mathbb{R}^3)^n \), as follows;

\[
e := p(\theta) - p_d(\xi).
\]

The shape Jacobian, \( J \in \mathbb{R}^{3n \times 3n} \), is defined by

\[
J(q) := \frac{\partial e}{\partial q},
\]

where \( q := [\theta^T \xi^T]^T \). We assume that the shape Jacobian is always non-singular. Note that, although both \( p \) and \( p_d \) are not free in \((\mathbb{R}^3)^n\), their difference, \( e \), can be regarded as an element of \((\mathbb{R}^3)^n\) in the vicinity of the origin \( e = 0 \) under this assumption. In other words, the neighborhood of \( e = 0 \) can be viewed as \((\mathbb{R}^3)^n\). For more details, see [8].

### 2.4 Serial-Chain Transformations

For any rotation acting on a rigid body, \( R_\sigma \in SO(3) \), define a serial-chain transformation, \( \sigma \), as follows;

\[
\sigma : (\mathbb{R}^3)^n \rightarrow (\mathbb{R}^3)^n
\]

\[
p \mapsto \Phi_\sigma p,
\]

or

\[
\sigma : SO(3)^n \rightarrow SO(3)^n
\]

\[
\Phi \mapsto \Phi_\sigma \Phi,
\]

where

\[
\Phi_\sigma := \text{block diag} \{R_\sigma \ R_\sigma \ \cdots \ R_\sigma\}.
\]

The above definition shows that a serial-chain transformation, \( \sigma \), acts on both elements of \((\mathbb{R}^3)^n\) and \(SO(3)^n\). Here we introduce the notion of congruence of manipulator’s shapes as follow;

**Definition 1 (Shape Congruence)** Two shapes of a manipulator, \( p_1, p_2 \in Q \) are said to be congruent if there exists a serial-chain transformation \( \Phi_\sigma \) such that

\[
p_2 = \Phi_\sigma p_1.
\]

From the above definition, we understand that \( \sigma \) is a transformation which preserves the shape of a manipulator.

The tangent action, \( T\sigma \), and its dual one, \( T^*\sigma \), are represented by

\[
T\sigma : TQ \rightarrow TQ
\]

\[
(p, v) \mapsto (\Phi_\sigma p, \Phi_\sigma v),
\]

\[
T^*\sigma : T^*Q \rightarrow T^*Q
\]

\[
(p, \Delta f) \mapsto (\Phi_\sigma p, \Phi_\sigma \Delta f).
\]

The projection of force distributions onto ones with respect to body frames, \( \pi_b^* \), is denoted by

\[
\pi_b^* : T^*Q \rightarrow (F_b)^n
\]

\[
(p, \Delta f) \mapsto \Phi_p^T \Delta f,
\]

where \((F_b)^n\) stands for the set of body-force distributions, and \( \Phi_p \in SO(3)^n \) is the following matrix defined by the link orientations, \( \Phi_1 \), corresponding to the shape, \( p \);

\[
\Phi_p := \text{block diag} \{\Phi_1 \ \Phi_2 \ \cdots \ \Phi_n\}.
\]

### 3 Whole-Arm Impedance

#### 3.1 Spatial Affinity

In this paper, we consider a compliance and damping as impedance elements. A compliance can be regarded as a conservative force field. Therefore, it is represented by a mapping of the shape space \( Q \) into the force-distribution space \( T^*Q \). On the other hand, a damping is a mapping of the shape-velocity space \( TQ \) into its dual space \( T^*Q \).

Many definitions of impedance for the whole arm of a serial-chain manipulator can be considered. For example, the simplest choice is setting the rotational impedances for each joint. However, this type of impedance is not geometrically natural, which we will see later in an example.

First, we review the definitions of spatial affinity of compliance and damping families originally proposed by Fasse and Broenink [3] which requires geometrically natural impedance:
Definition 2 (Spatial Affinity) Let \( C_{s,n} : Q \to TQ \) be a compliance mapping with parameters \( s \) and \( n \). A family of compliances, \( \{C_{s,n}\} \), is spatially affine if
\[
\pi_b^* \circ C_{\sigma(s),n} \circ \sigma = \pi_b^* \circ C_{s,n} ,
\]
independently of \( s, n \) and \( \sigma \). For a damping mapping \( D_{s',n'} : TQ \to T^*Q \) with parameters \( s' \) and \( n' \), a family of dampings, \( \{D_{s',n'}\} \), is spatially affine if
\[
\pi_b^* \circ D_{\sigma(s'),n'} \circ \sigma = \pi_b^* \circ D_{s',n'} ,
\]
independently of \( s', n' \) and \( \sigma \). The parameters \( s \) and \( s' \) are called spatial parameters, and \( n \) and \( n' \) are called non-spatial parameters.

Of course, in \( [3] \), \( \sigma \) means a rigid body transformation because the impedance is considered with respect to the tip position and orientation there. In this paper, we think \( \sigma \) as a serial-chain transformations defined in Section 2.4 instead of a rigid-body transformation. This spatial affinity of impedance ensures that congruent shapes have the same impedance properties.

### 3.2 Simple Extension

It seems natural to consider the following compliance for the whole arm;
\[
C_{s,n} : Q \to T^*Q \\
p \mapsto (p, \Delta f_C) ,
\]
where \( p \in Q \) is the virtual target shape of the manipulator, \( R_C \in SO(3n) \) is a constant special orthogonal matrix, \( K_C \in \mathbb{R}^{3n \times 3n} \) is a constant positive-definite and diagonal matrix, and \( \Delta f_C \) is the force distribution generated by the compliance. Let spatial parameters \( s = \{p_v, R_C\} \) and non-spatial parameters \( n = \{K_C\} \).

Corresponding to the above compliance, consider the following damping \( D_{s',n'} \);
\[
D_{s',n'} : TQ \to T^*Q \\
(p, v) \mapsto (p, \Delta f_D) ,
\]
where \( p \in Q \) is the virtual target shape of the manipulator, \( R_D \in SO(3n) \) is a constant special orthogonal matrix, \( K_D \in \mathbb{R}^{3n \times 3n} \) is a constant positive-definite and diagonal matrix, and \( \Delta f_D \) is the force distribution generated by the damping. Let spatial parameters \( s' = \{p_v, v_v\}, R_D \) and non-spatial parameters \( n' = \{K_D\} \).

These compliance and damping mappings are simple extensions of those proposed by Fasse et al. in \( [3] \). We can show that \( C_{s,n} \) and \( D_{s',n'} \) satisfy the equations (27) and (28) respectively. However, we have the following questions:

1. The virtual target shape \( p_v \) must be decided by a given task. However, it is very difficult to find proper \( p_v \) because \( p_v \) must be the element of a restricted region in \( (\mathbb{R}^3)^n \), \( Q \).

2. The generated force distribution by the compliance is restricted because the value \( (p - p_v) \) is not free in \( (\mathbb{R}^3)^n \) in general. This is inconsistent with the fact that a force distribution is an element of linear space \( T^*Q \).

### 3.3 Curve Utilization

We make small modifications for the compliance and damping by utilizing the shape by a curve, \( p_d \), instead of \( p_v \). Modify \( \Delta f_C \) and \( \Delta f_D \) as
\[
\Delta f_C = R_C K_C R_C^T (p - p_d) , \hspace{1cm} (33) \\
\Delta f_D = R_D K_D R_D^T (v - v_d) , \hspace{1cm} (34)
\]
where \( p_d \) is the shape defined by a curve \( c \) (see eq. (14)), and \( v_d \) is the time derivative of \( p_d \).

As mentioned at the end of Section 2.3, \( (p - p_d) \) is an element of \( (\mathbb{R}^3)^n \) in the vicinity of the origin. Therefore, the force distribution generated by the compliance is free in \( (\mathbb{R}^3)^n \) and has a meaning as an element of the force-distribution space \( T^*Q \). Besides, \( p_d \) is calculated from the desired curve \( c \) which can be decided by some reasonable way from measured environmental data. Note that we have new variables \( \xi \) because \( p_d \) is the function of \( \xi \) (see (14) again). It is better to control \( \xi \) positively so as to achieve the desired impedance performance. Then, we assume that the motion of \( \xi \) is governed by
\[
M_\xi \ddot{\xi} = f_\xi , \hspace{1cm} (35)
\]
where \( M_\xi \in \mathbb{R}^{n \times n} \) is a positive definite matrix which is a design parameter, and \( f_\xi \in \mathbb{R}^n \) is regarded as a force driving \( \xi \). By this motion model, \( p_d(\xi) \) is seen as the positions of \( n \) particles constrained in the desired curve, with underlying mass property \( M_\xi \), driven by the controlled force \( f_\xi \).

The potential function corresponding to \( C_{s,n} \) is described by
\[
U_{s,n} : Q \to \mathbb{R} \\
p \mapsto \frac{1}{2} \left\langle K_C R_C^T (p - p_d), R_C^T (p - p_d) \right\rangle , \hspace{1cm} (36)
\]
where $\langle \cdot , \cdot \rangle$ denotes the inner product of vectors in $(\mathbb{R}^3)^n$. On the other hand, the instantaneous power corresponding to $D_{s',n'}$ is described by

$$P_{\text{damp}} = \Delta f_D^T v = (v - v_d)^T R_D K_D R_D^T v . \quad (37)$$

We can also consider the instantaneous power by the motion of $p_d$ as follows:

$$P'_{\text{damp}} = \Delta f_D^T v_d = (v - v_d)^T R_D K_D R_D^T v_d . \quad (38)$$

We show that the modified compliance and damping are spatially affine.

**Theorem 1 (Spatial Affinity)** The family of compliance for the whole arm $\{C_{s,n}\}$ defined by (29) and (33) is spatially affine with respect to $s = \{p_d, R_C\}$, $n = \{K_C\}$ and $\sigma$. The family of damping for the whole arm $D_{s',n'}$ defined by (31) and (34) is spatially affine with respect to $s' = \{(p_d, v_d), R_D\}$, $n' = \{K_D\}$ and $\sigma$.

(proof) From the orthogonality of $R_C$, we can calculate $C_{\sigma(s),n} \circ \sigma$ as

$$C_{\sigma(s),n} \circ \sigma : Q \rightarrow T^*Q \quad p \mapsto (\Phi_{s}p, \Delta f_C|_{\sigma}) , \quad (39)$$

where

$$\Delta f_C|_{\sigma} = (\Phi_{s}R_C)^T K_C (\Phi_{s}R_C)^T (\Phi_{s}p - \Phi_{s}p_d) = \Phi_{s}\Delta f_C . \quad (40)$$

Thus, we have

$$\pi_b^s \circ C_{\sigma(s),n} \circ \sigma : Q \rightarrow (F_b)^n \quad p \mapsto (\Phi_{s}p)^T \Delta f_C|_{\sigma} \quad \Phi_{s}p = \Phi_{s}\Delta f_C \quad \pi_b^s \circ C_{s,n} , \quad (41)$$

which shows the spatial affinity of $C_{s,n}$.

The proof of the spatial affinity of $D_{s',n'}$ can be done in the same manner.

(Q.E.D.)

### 3.4 Control Torques

The control torques $\tau_C, \tau_D \in (\mathbb{R}^2)^n$ corresponding to $C_{s,n}, D_{s',n'}$ can be obtained by

$$\tau_C = -\frac{\partial U_{s,n}}{\partial \theta} , \quad P_{\text{damp}} = -\tau_D^T \dot{\theta} . \quad (42)$$

Therefore, the impedance torque $\tau := \tau_C + \tau_D$ is given by

$$\tau = -\frac{\partial p^T}{\partial \theta} \left\{ R_C K_C R_C^T (p - p_d) + R_D K_D R_D^T (v - v_d) \right\} . \quad (43)$$

We require that $\xi$ satisfies the similar relations for the compliance and damping forces exerted to $\xi$, i.e.,

$$f_{\xi,C} = -\frac{\partial U_{s,n}}{\partial \xi} , \quad P'_{\text{damp}} = -f_{\xi,D}^T \dot{\xi} . \quad (44)$$

Then, we obtain the following motion law for $\xi$

$$\ddot{\xi} = K_{\xi} \frac{\partial p_d}{\partial \xi} \left\{ R_C K_C R_C^T (p - p_d) + R_D K_D R_D^T (v - v_d) \right\} . \quad (45)$$

where $K_{\xi}$ is the inverse of $M_{\xi}$, i.e., $K_{\xi}$ is a positive definite design parameter.

### 4 Illustrative Examples

Consider a four-joints planar manipulator and a straight line as the desired shape depicted in Figure 1-a. By the control laws, (43) and (45), the manipulator can reach the desired straight line as shown in Figure 1-b. However, an obstacle (the small square in the figure) may interfere the task achievement. In the case that the manipulator has the rotational spring property for each joint, the manipulator shape is quite different from the desired straight line as seen in Figure 1-c. On the other hand, the proposed compliance minimizes the shape error between the manipulator and the desired straight line as shown in Figure 1-d. From this simple example, we can imagine that the proposed compliance generates a hammock-like enveloping property.

![Figure 1: Enveloping property](image-url)
in Figure 2, by putting the curve specifying the virtual target shape of the impedance inside of a grasped object (the soccer ball), the shape error between the shape of a manipulator (the human arm) \[ p = \left[ p_1^T \ p_2^T \ p_3^T \right]^T \] and the curve shape \[ p_d = \left[ c(\xi_1)^T \ c(\xi_2)^T \ c(\xi_3)^T \right]^T \] is defined. The shape error \[ p - p_d \] is expressed by the white arrows in the figure. The proposed compliance generates the force distribution roughly proportional to the shape error which gives the stiffness of the grasp. It should be noted that \( c(\xi_1), c(\xi_2), c(\xi_3) \) are not the fixed points, but moves along the curve according to the motion law (45). The motion law has a geometric meaning to drives particles on the curve such that each error vector (i.e., each white arrow) becomes perpendicular to the tangent vector of the curve. However, we need more rigorous analysis to ensure the stability and robustness of whole-arm grasping.

5 Conclusion

In this paper, we proposed a mechanical impedance for the whole arm. The proposed impedance has the geometrically natural property called spatial affinity and includes a spatial curve which describes a virtual reference of the impedance. We also derived the impedance torque together with the motion law of the curve parameters. We gave some simple examples to illustrate the meaning of the proposed impedance.

References