Decentralized Control of Multiple Mobile Robots
Transporting a Single Object in Coordination
without Using Force/Torque Sensors

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Abstract
In this paper, we propose a decentralized control system of multiple mobile robots transporting a single object in coordination without using force/torque sensors. In this system, each robot is controlled as if it has a specified impedance dynamics without using a force/torque sensor. The robots transport an object in coordination using a leader-follower type control algorithm. The effect of parameter identification errors is discussed and a method to reduce it is proposed. The proposed control algorithm is experimentally applied to two mobile robots, and the experimental results illustrate the validity of the proposed control algorithm.

1 Introduction
When we would like to move a large and heavy object which could not be handled by a person, we carry it in cooperation with other persons to reduce the load of each person. The coordination of multiple robots, as shown in Figure 1, has some advantages similar to such a human behavior.

Many control algorithms have been proposed for the handling of a single object by multiple robots in coordination [1]-[6] etc. Most of these control algorithms proposed so far have been designed under the assumption that the force/moment applied to the object is available. Therefore, each robot has to have a force/torque sensor to measure the force/moment applied to the object. The force/torque sensor might be broken by the excessive force/moment, when the robots handle a large and heavy object.

In this paper, we consider a problem of transporting a single object by multiple holonomic mobile robots in coordination and propose a decentralized control algorithm without using force/torque sensors. In the following part of this paper, we propose an alternative leader-follower type control algorithm based on the impedance control. We consider how to control the apparent impedance of each robot without using a force/torque sensor. The impedance control algorithm is designed based on the dynamics of each robot, which is assumed known a priori. We then consider the effect of parameter identification errors and propose a method to reduce it.

Finally, the proposed control algorithm is experimentally applied to two mobile robots, and the experimental results illustrate the validity of the proposed control algorithm.

2 Design of controller for each robot
2.1 Decentralized control algorithm
We have proposed the leader-follower type control algorithm for multiple holonomic mobile robots [7],[8]. In this algorithm, the motion command of the object is given to one of the robots, referred to as a leader. The other robots, referred to as followers, estimate the motion of the leader by themselves through the motion of the object and transport the object in coordination with the leader. This algorithm is designed based on velocity-servo controller, that is, the robots are assumed to move with commanded velocity. However, this algorithm ignores the dynamics of each robot and could not be applied to the leader-follower type control algorithm, which is based on the compliant motion control of each robot, without using a force/torque sensor. In this subsection, we consider
the leader-follower type control algorithm, based on the dynamics of each robot by extending the algorithm in [7],[8].

We assume that each robot is controlled around its steady-state position and velocity estimation errors, the desired trajectory of the leader. To eliminate the trajectory deviation of the representative point of the object corresponding to the motion of the $k$-th robot, $\Delta X_{ok}(k = l, k = i)$ is expressed as follows:

$$\Delta X_{ok}(k = l, k = i)$$

where $\Delta X_{ok} = [ \Delta x_{ok} \Delta \theta_{ok} ]^T \in R^3$ is the trajectory deviation of the representative point of the object corresponding to the motion of the $k$-th robot.

$$\Delta X_{cl} = X - X_{dl} \quad (3)$$

$$\Delta X_{ct} = X - X_{ct} \quad (4)$$

$kR$ is a vector defined by the element of the position vector $^k r_{cc} = [ r_x r_y ]^T$ as shown in [8]. $^k R$ is expressed as follows:

$$^k R = [ -r_y \ r_x ] \quad (5)$$

For the simplicity of explanation, we consider the case of transporting a single object by two mobile robots in coordination, that is $i = 1$.

Assuming that the external force/moment is not applied to the object, we obtain the following relations with respect to the force/moment $F_k^e$, $F_k^i$ with $k = l, k = i$.

$$F_k^e + F_k^i = 0 \quad (6)$$

$$F_k^i + F_k^i = 0 \quad (7)$$

In a steady-state, we obtain the following relation from eq.(1) with $k = l$, $k = 1$, eq.(6) and eq.(7).

$$\Delta X_l + \Delta X_1 = 0 \quad (8)$$

Subtracting eq.(2) with $k = l$ from eq.(2) with $k = 1$ and eliminating $X$, we obtain the following relation.

$$\Delta X_1 - \Delta X_l = X_{dl} - X_{ct} \quad (9)$$

Let $\Delta X_{dl}$ be the difference between the desired trajectory of the leader $X_{dl}$ and the estimated trajectory of the follower $X_{ct}$, with respect to the representative point of the object. From eq.(8) and eq.(9), $\Delta X_{dl}$ is expressed as follows:

$$\Delta X_{dl} = X_{dl} - X_{ct} = 2\Delta X_1 \quad (10)$$

It should be noted that the follower can calculate $\Delta X_{dl}$ by using its own trajectory deviation $\Delta X_1$.

Let us consider how the follower estimates the desired trajectory of the leader using $\Delta X_{dl}$ calculated by eq.(10). Let $G_1$ be the transfer function matrix to estimate $X_{dl}$, based on $\Delta X_{dl}$ as shown in Figure 3(a). From eq.(10), Figure 3(a) can be rewritten as a feedback system as shown in Figure 3(b). If $\Delta X_{dl}$ converges to zero, the follower can exactly estimate the desired trajectory of the leader. To eliminate the steady-state position and velocity estimation errors, the transfer function matrix $G_1$ is designed as follows:

$$G_1 = \frac{a_1 s + b_1}{s^2} I_3 \quad (11)$$

where $a_1 > 0$, $b_1 > 0$. $I_3$ is a $3 \times 3$ initial matrix.
2.2 Impedance control without using force/torque sensors

In this subsection, we consider a method that each robot is controlled as if it has a specified impedance dynamics without using a force/torque sensor, by using the method proposed in [10].

We assume that the dynamics of the k-th holonomic mobile robot is expressed as follows:

\[ F^e_k + F^n_k = M_k \ddot{X}_k + D_k \dot{X}_k + C_k \overline{sgn}(g(\dot{X}_k)) \]  
(12)

where \( F^e_k \in \mathbb{R}^3 \) is the force/moment applied to the k-th robot, \( F^n_k \in \mathbb{R}^3 \) is the force/moment applied to the k-th robot by the input torque \( \tau^a_k \in \mathbb{R}^n \) generated by actuators of the k-th robot. \( M_k, D_k \in \mathbb{R}^{3 \times 3} \) are the inertia matrix and the damping matrix of the k-th robot respectively, and \( C_k \in \mathbb{R}^{3 \times n} \) is the Coulomb friction matrix corresponding to friction of wheels. \( X_k \in \mathbb{R}^3 \) is the trajectory of the k-th robot, \( n \in \mathbb{R} \) is the number of the actuators which drive each robot. Let \( J \in \mathbb{R}^{3 \times n} \) be a \( 3 \times n \) Jacobian matrix relating the velocity of the k-th robot \( \dot{X}_k \) to the angular velocity \( \dot{\theta}^w_k \in \mathbb{R}^n \) of wheels of the k-th robot. \( \overline{sgn}(g(\dot{X}_k)) \) is expressed as follows:

\[ \overline{sgn}(g(\dot{X}_k)) = [\overline{sgn}(\dot{\theta}^w_{k_1}), \overline{sgn}(\dot{\theta}^w_{k_2}), ..., \overline{sgn}(\dot{\theta}^w_{k_n})]^T \]
(13)

\[ \overline{sgn}(\dot{\theta}^w_{k_i}) = \begin{cases} 1 & \dot{\theta}^w_{k_i} \geq 0 \\ -1 & \dot{\theta}^w_{k_i} < 0 \end{cases} \]
(14)

\[ g(\dot{X}_k) = \dot{\theta}^w_k = J^{-1} \dot{X}_k \]
(15)

where \( \dot{\theta}^w_k = (\dot{\theta}^w_{k_1}, \dot{\theta}^w_{k_2}, ..., \dot{\theta}^w_{k_n}) \in \mathbb{R}^n \) is a vector, which consists of desired angles of wheels of the k-th robot.

The relation between the input torque \( \tau^a_k \) and the force/moment \( F^a_k \) applied to the k-th robot by \( \tau^a_k \) is expressed as follows:

\[ \tau^a_k = J^T F^a_k \]
(16)

We consider the following input torque vector \( \tau^a_k \) to the k-th robot.

\[ \tau^a_k = J^T M_k (J \dot{\theta}^w_{k_d} + J \dot{\tilde{\theta}}^w_{k_d}) + J^T D_k \dot{\theta}^w_k + J^T C_k \overline{sgn}(\dot{\theta}^w_k) - J^T F^i_k \]
(17)

where \( \theta^w_{k_d} \in \mathbb{R}^n \) is a vector, which consists of desired angles of the wheels of the k-th robot.

Then from eq.(12), eq.(16) and eq.(17), each robot is controlled as if it has the impedance dynamics as follows:

\[ M_k \Delta \ddot{X}_k + D_0 \Delta \dot{X}_k + K_0 \Delta X_k = F^e_k - F^i_k \]
(18)

It should be noted that this equation has the same form as eq.(1). It should be also noted that we do not use the acceleration feedback which is included the method proposed in [10].

Thus, each robot could be controlled as if it has the specified impedance dynamics without using a force/torque sensor, by the control law \( \tau^a_k \) expressed by eq.(17), and the multiple mobile robots could transport a single object without using force/torque sensors in coordination by applying the leader-follower type control algorithm discussed in subsection 2.1.

3 Parameter identification errors

If the parameters of each robot \( M_k, D_k, C_k \) are identified precisely, each robot could be controlled as if it has the specified impedance dynamics without using a force/torque sensor and multiple mobile robots could transport a single object in coordination without using force/torque sensors. However, the parameters of each robot could not be identified precisely. For example, the Coulomb friction matrix changes based on the condition of the ground.

In the following part of this section, we first consider how the parameter identification errors influence the motion of each robot, then propose a method to reduce the effect of parameter identification errors.

3.1 Effect of parameter identification errors

Assuming that the real parameters of each robot \( M_k, D_k, C_k \) are identified as \( \hat{M}_k, \hat{D}_k, \hat{C}_k \), the input torque vector \( \tau^a_k \) is expressed as follows:

\[ \tau^a_k = J^T \hat{M}_k (J \dot{\theta}^w_{k_d} + J \dot{\tilde{\theta}}^w_{k_d}) + J^T \hat{D}_k \dot{\theta}^w_k + J^T \hat{C}_k \overline{sgn}(\dot{\theta}^w_k) - J^T F^i_k \]

\[ + J^T D_0 (\dot{\theta}^w_{k_d} - \dot{\tilde{\theta}}^w_{k_d}) + J^T K_0 (\theta^w_{k_d} - \theta^w_k) \]
(19)

From eq.(12), eq.(16) and eq.(19), each robot is controlled as if it has the impedance dynamics as follows:

\[ (F^e_k - F^i_k) + F^p_k = M_k \Delta \ddot{X}_k + D_0 \Delta \dot{X}_k + K_0 \Delta X_k \]
(20)

where \( F^p_k \in \mathbb{R}^3 \) is the equivalent disturbance force/moment corresponding to the parameter identification errors of the k-th robot. \( F^p_k \) is expressed as follows:

\[ F^p_k = \Delta M_k \dot{X}_kd + \Delta D_k \dot{X}_k + \Delta C_k \overline{sgn}(\dot{X}_k) \]
(21)

where \( \Delta M_k = \hat{M}_k - M_k (\in \mathbb{R}^{3 \times 3}) \), \( \Delta D_k = \hat{D}_k - D_k (\in \mathbb{R}^{3 \times n}) \), \( \Delta C_k = \hat{C}_k - C_k (\in \mathbb{R}^{3 \times n}) \). \( \dot{X}_kd \) is the desired trajectory of the k-th robot.
3.2 Method to reduce the effect of parameter identification errors

In this subsection, we propose a method to reduce the effect of the equivalent disturbance force/moment by extending the algorithm proposed in [9].

In subsection 3.1, we derive the equivalent disturbance force/moment which is applied to a robot corresponding to its own parameter identification errors. When multiple robots transport a single object in coordination, the motion of each robot is also affected by the parameter identification errors of the other robots and the object. Note that the effect of the object is included in the parameter identification errors of each robot.

Let \( F_{\text{ext}}^k \in \mathbb{R}^3 \) be the resultant equivalent disturbance force/moment applied to the \( k \)-th robot. Then, \( F_{\text{ext}}^k \) is expressed as follows;

\[
F_{\text{ext}}^k = F_{\text{p}}^k + \hat{F}_{\text{p}}^k \tag{22}
\]

where \( F_{\text{p}}^k \) is the equivalent disturbance force/moment applied to the \( k \)-th robot corresponding to its own parameter identification errors, and \( \hat{F}_{\text{p}}^k \in \mathbb{R}^3 \) is the resultant equivalent disturbance force/moment applied to the \( k \)-th robot by the other robots.

Let \( \Delta X_{\text{ext}}^k, \Delta X_{\text{ext}}^l \in \mathbb{R}^3 \) be the trajectory deviations of the leader and the follower respectively, which are caused by the resultant equivalent disturbance force/moment \( F_{\text{ext}}^k, F_{\text{ext}}^l \). Since each robot is controlled to have the dynamics of eq.(20), the effect of the resultant equivalent disturbance force/moment \( F_{\text{ext}}^k \) on the motion deviation of each robot is expressed as follows.

\[
M_l \Delta \ddot{X}_{\text{ext}}^l + D_l \dot{\Delta} X_{\text{ext}}^l + K_l \Delta X_{\text{ext}}^l = F_{\text{ext}}^l \tag{23}
\]

\[
M_1 \Delta \ddot{X}_{\text{ext}}^l + D_0 \dot{\Delta} X_{\text{ext}}^l + K_0 \Delta X_{\text{ext}}^l = F_{\text{ext}}^l \tag{24}
\]

When the parameters of each robot are not identified precisely and the control input \( \tau_{\text{e}}^l \) of eq.(17) is calculated with parameter identification errors, \( F_{\text{ext}}^l \) of eq.(1) also includes the effect of these errors. Subtracting eq.(23) and eq.(24) from eq.(1) with \( k = l, k = 1 \), we obtain the following relations.

\[
M_l ( \Delta \ddot{X}_l - \Delta \ddot{X}_{\text{ext}}^l ) + D_{0l} ( \dot{\Delta} X_l - \dot{\Delta} X_{\text{ext}}^l ) + K_{0l} ( \Delta X_l - \Delta X_{\text{ext}}^l ) = ( F_{\text{p}}^l + \hat{F}_{\text{p}}^l ) - F_{\text{ext}}^{\text{in}} \tag{25}
\]

\[
M_1 ( \Delta \ddot{X}_l - \Delta \ddot{X}_{\text{ext}}^l ) + D_0 ( \dot{\Delta} X_l - \dot{\Delta} X_{\text{ext}}^l ) + K_0 ( \Delta X_l - \Delta X_{\text{ext}}^l ) = ( F_{\text{p}}^l + \hat{F}_{\text{p}}^l ) - F_{\text{ext}}^{\text{in}} \tag{26}
\]

Eq.(25) and eq.(26) express the dynamics of each robot as shown in eq.(1) with \( k = l, k = 1 \) without the effect of parameter identification errors. Similar to eq.(8), we obtain the following relation.

\[
( \Delta X_l - \Delta X_{\text{ext}}^l ) + ( \Delta X_l - \Delta X_{1\text{ext}}^l ) = 0 \tag{27}
\]

Eliminating the effect of the resultant equivalent disturbance force/moment applied to each robot from eq.(2), we obtain the following relations.

\[
\Delta x_k - \Delta x_{\text{ext}}^k = ( \Delta x_{\text{ok}} - \Delta x_{\text{ext}}^k )
- ( \Delta \theta_{\text{ok}} - \Delta \theta_{\text{ext}}^k ) k R^T \tag{28}
\]

\[
\Delta \theta_k - \Delta \theta_{\text{ext}}^k = \Delta \theta_{\text{ok}} - \Delta \theta_{\text{ext}}^k \tag{29}
\]

where \( \Delta X_{\text{ext}}^k = [ \Delta x_{\text{ext}}^k \Delta \theta_{\text{ext}}^k ]^T \in \mathbb{R}^3 \). \( \Delta X_{\text{ext}}^k \) is the trajectory deviation of the representative point corresponding to the motion caused by the resultant equivalent disturbance force/moment applied to each robot. \( \Delta X_{\text{ext}}^k \) satisfies the following relation as shown in [9].

\[
( \Delta x_{\text{ok}} - \Delta x_{\text{ext}}^k ) + ( \Delta X_{1} - \Delta X_{\text{ext}}^k ) = 0 \tag{30}
\]

Subtracting eq.(28) and eq.(29) with \( k = l \) from eq.(28) and eq.(29) with \( k = 1 \) and eliminating \( X = [ x \ \theta ]^T \), by using eq.(3),eq.(4),eq.(27),eq.(30), and eq.(31), we have the trajectory estimation error \( \Delta X_{d1} \) as follows;

\[
\Delta X_{d1} = X_{d1} - X_{e1} = 2 ( \Delta X_1 - \Delta X_{\text{ext}}^1 ) \tag{32}
\]

Eq.(32) expresses the trajectory estimation error without the effect of the resultant equivalent disturbance force/moment. Therefore, the follower can estimate the desired trajectory given to the leader, reducing the effect of the resultant equivalent disturbance force/moment, by using eq.(32) instead of eq.(10). The follower can calculate \( \Delta X_1 \) by using its own information, because \( \Delta X_1 \) is the trajectory deviation of the follower itself.

Let us consider how to calculate \( \Delta X_{\text{ext}}^1 \). From eq.(30) and eq.(31), \( \Delta X_{\text{ext}}^1 \) is expressed as follows;

\[
\Delta X_{\text{ext}}^1 = ( \Delta x_{\text{ok}} + \Delta x_{\text{ext}}^1 ) / 2 \tag{33}
\]

From eq.(2)–eq.(4) and eq.(33), \( \Delta X_{\text{ext}}^1 \) is expressed as follows;

\[
\Delta X_{\text{ext}}^1 = [ \Delta x_{\text{ext}}^1 - \Delta \theta_{\text{ext}}^1 ]^T \tag{34}
\]

where

\[
\Delta x_{\text{ext}}^1 = x - ( x_{dl} + x_{e1} ) / 2 \tag{35}
\]

\[
\Delta \theta_{\text{ext}}^1 = \theta - ( \theta_{dl} + \theta_{e1} ) / 2 \tag{36}
\]

The leader knows its own desired trajectory \( X_{dl} = [ x_{dl} \ \theta_{dl} ]^T \). Since the leader could calculate \( x_{e1} = [ x_{e1} \ \theta_{e1} ]^T \) from the current position and orientation of the representative point of the object using geometric relations between the leader and the follower,
the leader can calculate $\Delta X_{1}^{ext}$ from eq.(34), eq.(35) and eq.(36). For the calculation of $X_{e1} = [x_{e1}, \theta_{e1}]^{T}$ from its current state of the representative point of the object, refer to [9].

If the leader broadcasts $\Delta X_{1}^{ext}$ to the follower, the follower can obtain $\Delta X_{1}^{ext}$ and calculate the trajectory estimation error $\Delta X_{d1}$ by using eq.(32).

By using the trajectory estimation error $\Delta X_{d1}$ and the transfer function matrix $G_{1}$ which we design in subsection 2.1, the follower could estimate the desired trajectory of the leader with reducing the effect of the resultant equivalent disturbance force/moment caused by parameter identification errors.

It is sufficient to broadcast the information of the equivalent disturbance force/moment $\Delta X_{1}^{ext}$ at an interval which is longer than a sampling interval, because the resultant equivalent disturbance force/moment caused by parameter identification errors changes smoothly.

In the case of transporting a single object by multiple mobile robots more than two, each follower can estimate the desired trajectory of the object by using the concept of the virtual leader proposed in [7],[8]. Thus, multiple mobile robots can transport a single object in coordination without using force/torque sensors.

4 Experiments
The proposed control algorithm was experimentally applied to two omni-directional mobile robots, ZEN, developed by RIKEN [11], shown in Figure 4. Each mobile robot has three degrees of freedom of motion. The control algorithm is implemented using VxWorks.

First, we identified the parameters of each robot $M_{k}$, $D_{k}$, $C_{k}$ by using the relation between the input torques and the resultant motion of each robot. We controlled each robot by using the identified parameters. In the experiment, the leader was given the desired trajectory along y axis from $0[m]$ to $1.5[m]$ for $20[sec]$, which was calculated by a fifth order polynomial of time and the orientations of all of the robots were kept constant during the transportation of the object. The follower estimated the desired trajectory of the leader using the algorithm proposed in this paper.

The experimental results are shown in Figure 5, which is described with respect to the coordinate frame of the object. The solid lines show the results of the leader. The dotted lines show the results of the follower. You can see that the leader and the follower transported the object successfully without using force/torque sensors. In this experiment, $\Delta X_{1}^{ext}$ was calculated by the leader and broadcasted to the follower every $100\msec$ although the sampling interval of each controller was $1\msec$.

Figure 7 shows an example of the experiments. In this figure, the desired trajectory of the object was commanded to the leader as shown in Figure 6.

5 Conclusions
In this paper, we proposed a decentralized control algorithm of multiple mobile robots transporting a single object in coordination without using force/torque sensors. The proposed control algorithm was experimentally applied to two omni-directional mobile robots, and the experimental results illustrated the validity of the proposed control algorithm.

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References
Figure 6: Desired trajectory in Figure 7


Figure 7: Example of Experiments