Control of a Suspended Load Using Inertia Rotors with Traveling Disturbance

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Abstract

Rotational control and swing suppression of a crane suspended load model with traveling disturbance are studied. A rotational free rigid body suspended by a single rope is controlled using three inertia rotors. The end-supporting-point of the single rope is forced to be traveled as disturbance. Control angles and angular velocities are derived from measured data of fiber optic gyros installed on the suspended load. Rotor angular velocities controlling the suspended load are obtained by integrating the computed digital sliding-mode control feedback accelerations based on the coupling system's dynamics. Experiments and simulations investigate simultaneous control of the load's rotational orientation and swing suppression for traveling disturbance.

1. Introduction

A crane suspended load in a construction work is frequently rotated and swung by wind pressure or inertia force accompanied with movement of the crane. Most studies to control the suspended load have focused on suppressing swing of the load as a lumped mass and manipulating the crane. For example, Sakawa and Nakazumi [1] treated a rotary crane and Lee [2] discussed the simultaneous traveling and traversing trolley. The suspended load is actually a rigid body different from above-mentioned a lumped mass model and possible to be rotated. Kanki et al. [3] developed an active control device using gyroscopic moment to control the rotation of a crane suspended load.

There are few studies to control both rotation and swing of the suspended load as a rotational free rigid body. Yoshida and Mori [4] studied simultaneous control of rotation and swing of a pendulum having a rotational free rigid body using inertia rotors and recently Yoshida and Yajima [5] studied for a rotational free rigid body model suspended by a single rope with initial swing disturbance.

This paper presents rotational control and swing suppression of a suspended load model with traveling disturbance. The rotational free rigid body is suspended by a single rope and controlled using three inertia rotors, where the end-supporting-point of the single rope is forced to be traveled as disturbance. The suspended load as a rigid body has non-actuating (passive) three-degree-of-freedom and three inertia rotors have individually actuating (active) one-degree-of-freedom. The model system has six-degree-of-freedom and passive degrees of freedom are controlled by active degrees of freedom based on the coupling system's dynamics.

2. Dynamic Model and Controller

Figure 1 shows static state of the suspended load model, where the coordinate \( \hat{x}_0 \hat{y}_0 \hat{z}_0 \) is nonmoving base frame and \( \hat{x}' \hat{y}' \hat{z}' \) is moving frame fixed with the load. The rope-end-point travels to \( \hat{x}_0 \) direction. Three inertia rotors installed on the load. The purpose of this study is to control the motion of the load using inertia rotors. Fig.2 shows moving state, where the suspended load is represented with the rope-end-point displacement \( x, \ zxy \) Euler angles \( \theta_1, \theta_2, \theta_3 \) and \( zxz \) Euler angles \( \bar{\theta}_1, \bar{\theta}_2, \bar{\theta}_3 \). Angular velocities of the load are \( \omega_1, \omega_2, \omega_3 \) with respect to \( \hat{x}, \hat{y} \) and \( \hat{z} \) axes.

Angles and angular velocities of inertia rotors are
\[ \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3 \text{ and } \omega_1, + \dot{\theta}_1, \omega_4 + \dot{\theta}_4, \text{ and torques are } \tau_1, \tau_2, \tau_3 \text{ with respect to } \hat{z}, \hat{x} \text{ and } \hat{y} \text{ axes.} \]

Based on XYZ Euler angles \( \theta_1, \theta_2, \theta_3 \) of the suspended load and \( \theta_4, \theta_5, \theta_6 \) of inertia rotors, the equation of motion can be written as

\[
\begin{bmatrix}
M_{cc} & M_{cf} \\
M_{fc} & M_{ff}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_c \\
\ddot{q}_f
\end{bmatrix} + \begin{bmatrix}
h_c \\
h_f
\end{bmatrix} + \begin{bmatrix}
X_c \\
X_f
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix},
\]

(1)

where

\[
M_{cc} = \begin{bmatrix}
m_{11} & m_{12} & m_{13} \\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{bmatrix},
M_{cf} = \begin{bmatrix}
m_{14} & m_{15} & m_{16} \\
m_{24} & m_{25} & m_{26} \\
m_{34} & m_{35} & m_{36}
\end{bmatrix},
M_{fc} = M_{cc}^T,
M_{ff} = \begin{bmatrix}
I_{k1} & 0 & 0 \\
0 & I_{k1} & 0 \\
0 & 0 & I_{k2}
\end{bmatrix},
\]

\[
q_c = [\theta_1, \theta_2, \theta_3]^T, \quad q_f = [\theta_4, \theta_5, \theta_6]^T.
\]

\[
h_c = [h_1, h_2, h_3]^T, \quad h_f = [h_4, h_5, h_6]^T, \quad X = [x_1, x_2, x_3]^T,
\]

\[
\bar{\theta} = [0, 0, 0]^T, \quad \bar{\tau} = [\tau_1, \tau_2, \tau_3]^T.
\]

\( q_c \) shows controlled (non-actuating) angle state vector and \( q_f \) free (actuating) angle state vector. Mass matrix \( M_{cc}, M_{cf} (= M_{fc}) \) are function of \( \theta_2, \theta_3 \), \( h_c, h_f \) are vector of Coriolis, centrifugal and gravity terms and function of \( \theta_2, \theta_3, \theta_i (i = 1 \sim 6) \). \( X \) is vector of traveling disturbance and function of \( \theta_1, \theta_2, \theta_3 \).

The system equation of controlled XYZ Euler angle’s state vector \( q_c \) is linearized by using the control input vector \( u_c \) as

\[
\ddot{q}_c = u_c = [u_1, u_2, u_3]^T.
\]

(2)

A digital sliding-mode controller by which the controlled angles \( \theta_1, \theta_2, \theta_3 \) of the suspended load follow the desired values \( \theta_{1d}, \theta_{2d}, \theta_{3d} \) is designed from equation (2). Digital state equation of control angle error is obtained using sampling time \( T \) and time steps \( k (=1, 2, \cdots) \) as

\[
E_{i,k+1} = pE_{i,k} - q \bar{\tau}_{ik} (i = 1 \sim 3)
\]

(3)

\[
E_{i,k} = \begin{bmatrix}
e_a \\
e_{\bar{\theta}}
\end{bmatrix}, \quad p = \begin{bmatrix}
1 & T \\
0 & 1
\end{bmatrix}, \quad q = \begin{bmatrix}
0.5T^2 \\
T
\end{bmatrix}
\]

Fig.1 Suspended load model using inertia rotors with traveling disturbance.

Fig.2 Coordinates and angles of the suspended load model.
Switching function $\sigma_{ik}$ is defined with scalar vector $S = [s_1, s_2]$ as follows
\[ \sigma_{ik} = SE_{ik} \quad (i = 1 \sim 3) \]  
where, $\sigma_{ik} = 0$ gives switching line. Sliding-mode control input $\pi_{ik}$ can be represented as follows, where $\pi_{eqik}$ is equivalent input on switching line and $\pi_{nik}$ is nonlinear input acting to reach switching line,
\[ \pi_{ik} = \pi_{eqik} + \pi_{nik}. \]  

From equation (2) to (5), sliding-mode controller is taken as
\[ u_{eqik} = (S_q)^{-1}S(p-I)E_{ik}, \]
\[ u_{nik} = \eta(Sq)^{-1}\sigma_{ik} \quad for \ 0 < \eta < 2. \]

After putting equation (2), (7) into equation (1), upper part of the equation of motion (1) becomes as
\[ M_vu_v + M_q\ddot{q}_f + h_v + X\ddot{\theta} = 0. \]

Above equation (8) shows the dynamic coupling between controlled angles $q_v$ and free angles $q_f$. Then, estimated accelerations $\hat{u}_f$ are given as
\[ \hat{u}_f = -M_{gf}^{-1}(M_vu_v + h_v + X\ddot{\theta}). \]

By integrating equation (9) with the sampling time $T$, manipulating velocities of inertia rotors $\dot{q}_f = \ddot{v}_k = [v_{4k} \ v_{5k} \ v_{6k}]^T$ are obtained as
\[ \ddot{v}_k = v_{4k} + \ddot{u}_f T. \]  

Fiber optic gyros installed on the suspended load can measure the angles $\theta_1, \theta_2, \theta_3$ and the angular velocities $\omega_x, \omega_y, \omega_z$. $zxy$ Euler angles $\theta_1, \theta_2, \theta_3$ and $zxz$ Euler angles $\theta_1, \theta_2, \theta_3$ are mutually convertible. $zxz$ Euler angles are obtained by measured angles as
\[ \ddot{\theta}_i = \ddot{\theta}_i - \ddot{\theta}_i, \]
\[ \ddot{\theta}_i = \cos^{-1}(C_i, C_i), \]
\[ \ddot{\theta}_i = A \tan(2(-C_i, S_i, S_i)). \]

Under the condition that $\theta_1, \theta_2, \theta_3$ are directly measured by fiber optic gyros, $\theta_i$ must be calculated based on equation (11) as
\[ \theta_i = A \tan2(C_i, S_i) + S_i, \]  
where $C_i$ and $S_i$ are defined as $\cos \theta_i$ and $\sin \theta_i$ for $i = 1, 2, 3$, respectively.

Angular velocities $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ are obtained using $\omega_x, \omega_y, \omega_z$ and $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$ as
\[ \dot{\theta}_i = \begin{bmatrix} -S_i/C_i; & C_i/C_i; & \omega_x \\ & & \omega_y \\ & & \omega_z \end{bmatrix}, \]  
where $T_2$ is $\tan \theta_2$. Fig.3 shows the control system.

Fig.3 Control system.

Simulation is performed for the sampling time $T$ between $t (=step k)$ and $t+T (=step k+1)$. Following simultaneous differential equations are solved using Runge-Kutta numerical integration.
\[ \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} = - M_{gf}^{-1}(M_vu_v + h_v + X\ddot{\theta}), \]
\[ \ddot{\theta}_f = (\ddot{v}_k - \ddot{v}_{4k})/T. \]  

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Since the manipulating velocities are held and constant at the step $k$, angular accelerations of free angle $\dot{q}_f$ are zeros in the time between sampling time $T$. However, the manipulating velocities change at each time step and therefore accelerations of free angle are approximately taken as $\ddot{q}_f$ that is the change of the manipulating velocity between $k-1$ and $k$ steps. $\ddot{q}_f$ assures the dynamic coupling in the simulation.

3. Experimental Results

Fig.4 shows the experimental suspended model photograph. Length between support point and the load's center of gravity is $l = 1.22m$, mass of the load including inertia rotors $m = 20.9kg$, moments of inertia of the load $I_1 = 1.36kgm^2$, $I_2 = 0.676kgm^2$, $I_3 = 1.17kgm^2$ and inertia rotors $I_{R1} = I_{R2} = I_{R3} = 0.0377kgm^2$. Maximum angular velocities of servomotors driving inertia rotors are $\dot{\theta}_1 = 173rpm$, $\dot{\theta}_2 = 184rpm$, $\dot{\theta}_3 = 168rpm$. Sliding-mode digital gains are $K_i = 80$, $K_z = 16$ using sampling time $T = 0.05sec$ and $\eta = 0.5$, $s_2 / s_1 = 0.1$, selected from simulation results using $\eta = 0 \sim 2$, $s_2 / s_1 = 0 \sim 10$. And, digital poles of closed system obtained from equation (3) and (6) using these digital gains are $\zeta = 0.5$ and $0.6$, therefore the control system is stable. Simulation is used by Runge-Kutta numerical integration in time by steps of size $0.005sec$. Fig.5 shows the linear traveling of the end-supporting-point of the single rope as disturbance for the suspended load. Fig.5(a) is traveling displacement moving 0.25m within 1.5 sec. Fig.5(b) shows velocity and acceleration, that is, accelerating to 0.5 sec, uniform velocity to 1.3 sec and decelerating to 1.5 sec. Initial angles of the suspended load are manually positioned and it is difficult to give the accurate same initial angles to the experiments of both without and with controls. But the differences of initial $\phi \chi \psi$ Euler angles are small between $\theta_{i1} = 25^\circ$, $\theta_{i2} = 1^\circ$, $\theta_{i3} = 0^\circ$ in the case of experimental values without control and $\theta_i = 27^\circ$, $\theta_{i2} = 0^\circ$, $\theta_{i3} = 0^\circ$ with control. Zero desired values $\theta_{i1}, \theta_{i2}, \theta_{i3}$ are given for controlled angle $\theta_{d1}, \theta_{d2}, \theta_{d3}$.

Fig.4 Experimental suspended model’s photograph.

![Fig.4 Experimental suspended model’s photograph.](image-url)
Fig. 6 shows the experimental time response of controlled rotational angle $\theta_1$ within 30 sec. Thin line indicates the case of without control and thick line with control, and the following figures show in similar ways. Thin line of without control moves from initial angle $25^\circ$ to maximum $-66^\circ$ at 12 sec and thereafter largely fluctuates. This phenomenon is explained by Yoshida and Mori [5] that both centrifugal force caused by swing and the difference between moment of inertia $I_x$ and $I_y$ generate yaw rotation torque. Thick line of with control moves rapidly from initial angle to zero desired value within 2 sec and hereafter keep desired value.

Fig. 7 shows the experimental swing angle’s components. Fig. 7(a) shows controlled angle $\theta_2$. Thin line of without control enlarges vibration amplitude to maximum $6^\circ$ between 5 sec to 15 sec and and thick line with control keeps within $1.5^\circ$ value from initial time to 30 sec. Fig. 7(b) shows controlled angle $\theta_3$. Both cases of without and with control are same till the time 15 sec, but after that time thin line of without control remains $4^\circ$ constant vibration amplitude and thick line with control damps to zero desired value.

Fig. 8 shows the response of swing angle ($\xi_{xz}$ Euler angle) $\bar{\theta}_2$. The amplitudes in the case of with control are suppressed under half of those of without control.

Fig. 9 shows swing displacement $x, y$ for the fixed coordinate when the load is at rest. The vibration displacement $x$ is induced by travel disturbance.

Thin line of $x$ without control leaves vibration amplitude at 30 sec, but thick line of $x$ with control damps to zero desired value.
Fig. 10 shows simulation time response of rotational angle $\theta_1$ corresponding to experimental result of Fig. 6. Torsional spring coefficient of the rope is considered as $0.1 \text{Nm/rad}$ in simulation. Fluctuation tendency of thin line without control shows the same pattern of experiment and thick line with control moves rapidly from initial angle to zero value same as experiment.

Fig. 11 shows simulation swing displacements $x, y$ corresponding to experimental result of Fig. 9 and these are similar as experimental ones.

4. Conclusions

A rotational free rigid body suspended by a single rope model with traveling disturbance is controlled using three inertia rotors. Velocity-command-type control system is developed for inertia rotors by integrating the computed feedback accelerations of digital sliding-mode control, based on the coupling system's dynamics. The experimental and simulation results show that inertia rotors can control the rotation and swing of the suspended load for traveling disturbance.

References