Enhanced Contour Control of SCARA Robot Under Torque Saturation Constraint

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Abstract—Contour control of a robot arm is an act of the end-effector tip being moved along a reference Cartesian path, with an assigned velocity. When torque saturation occurs, and lasts for some time, contour deteriorations result. In addition, system delay dynamics also causes contour deteriorations. In this paper, an offline trajectory generation algorithm is described, which achieves both optimum avoidance of torque saturation and delay dynamics compensation.

Index Terms—Contour control, feedforward compensator, industrial robot arm, offline trajectory generation, torque saturation.

I. INTRODUCTION

The main objective of this paper is to enhance contour control performance of a selective compliance assembly robot arm (SCARA) under torque constraint. Its implementation on a SCARA AR-M270 (Hirata Company Ltd.) robot arm is also presented. Contour control is a twofold task. It involves offline trajectory generation and online trajectory tracking. Trajectory generation starts with trajectory segmentation, in which a set of “way-points” is declared. Trajectory segments are approximated with either straight lines and/or some functions as proposed by Lin et al. [1]. Finally, the segments are linked in a sequence, which forms the generated trajectory. Trajectory tracking is implemented by the servo drive.

Research on contour control of serial link robot arms dates back to the 1950s. Whitney [2] proposed forward and reverse Jacobian transformation for velocity programming in straight-line path tracking. Paul [3] assigned knot points in real time. However, these could not apply in industry due to the inherent vulnerability to disturbances and the excessive computation burden involved in real time. Taylor [4] proposed an offline path planning with enough knot points so that the kinematic and dynamic constraints are satisfied. Luh et al. [5] derived time schedules for velocity and acceleration to minimize tracking time.

There are two major causes of contour deteriorations, namely, torque saturation and system delay dynamics. When torque saturation takes place, desired and actual joint dynamics become significantly different. These differences transform into Cartesian space as contouring deteriorations. System delay dynamics cause contour deteriorations, usually in the form of overshoots at trajectory corners. There is no widely accepted discipline in the industry for contouring operation of robot arms. Instead, contour deteriorations are partially compensated by expert practitioners, by incorporation of various ad hoc methods.

In this paper, an application-oriented trajectory generation methodology is presented for contour control of industrial robot arms. Both Cartesian and joint constraints are addressed, referring to actual industrial applications. It has been proven from experimental results that this method would significantly improve contour control performance.

II. SYSTEM ARCHITECTURE OF AN INDUSTRIAL ROBOT ARM

A. Overview of the Robot Arm System

A SCARA system is shown in Fig. 1(a). A reference input generator generates the sampled Cartesian trajectory and transforms it into sampled joint trajectories. Sampled joint trajectories are written onto the servo controller in real-time operation. The servo controller reads joint trajectory data and actuates joint motors accordingly. According to Fig. 1(a), in the two-degree-of-freedom configuration, robot arm parameters are \( l_1 = 0.27 \text{ m}, l_2 = 0.24 \text{ m}, m_1 = 20 \text{ kg}, \) and \( m_2 = 15 \text{ kg}, \) where \( m_i \) and \( l_i \) denote the mass and length of the \( i \)th link, respectively. Referring to Fig. 1(b), the robot arm kinematics that transform Cartesian coordinates to joint coordinates are given by

\[
\begin{align*}
x &= l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\
y &= l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2). 
\end{align*}
\]

Corresponding inverse kinematics are given by

\[
\begin{align*}
\theta_1 &= \sin^{-1}\left(\frac{y}{\sqrt{x^2 + y^2}}\right) - \sin^{-1}\left(\frac{l_2 \sin \theta_2}{\sqrt{x^2 + y^2}}\right) \\
\theta_2 &= \pm \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2l_1 l_2}\right)
\end{align*}
\]

B. Robot Arm Dynamic Model

It is assumed that each joint operates independently and coupling between links is neglected. Also neglected are the Coriolis and centripetal torques, which cause significant contributions to contouring performance only at high speeds [6]. In this paper, we do not address the contouring problem at high speeds, but contouring at normal speeds. Actually, under low-speed operation, which is below about 1/20 of the rated motor speed, torque saturation hardly occurs, even at sharp trajectory corners. On the other hand, under high-speed operation, above about 1/5 of the rated motor speed, more precise joint dynamics described by either Lagrange/Euler or Newton/Euler formulations should be used. However, industrial robot arms are not usually operated at such high speeds for contouring applications. Under a normal speed condition which is below about 1/5 of the rated motor speed, joint dynamics could be described by the second-order linear model, using only kinematics, as shown in Fig. 2. Under the same condition, however, torque saturation at sharp trajectory corners remains a significant problem. Under
normal speed. Coriolis and centripetal torque disturbances can be neglected, whereas gravity torques do not exist as the SCARA has only vertical axis joints. It is also assumed that the joint torque and acceleration are proportionally related as given by $(r_i, \tau_2)^T \propto (\dot{\theta}_i, \ddot{\theta}_2)^T$, where $i = 1, 2$ represents the torque of the $i$th joint motor. This relationship has been used extensively in modeling of contouring robot arms [6] and, consequently, torque saturation phenomena can be reasonably modeled by proportional acceleration saturation. The corresponding joint dynamic model of industrial robot arms is shown in Fig. 2. The mathematical expression of joint dynamics is given by

$$\ddot{\theta}_i(t) = \text{sat} \left[ K_{vi} \left( K_{pi} U_i(t) - \dot{\theta}_i(t) - \dot{\theta}_i(t) \right) \right]$$

$$\text{sat}(z) = \begin{cases} \underline{\dot{\theta}}_{i,\text{max}}, & z > \underline{\dot{\theta}}_{i,\text{max}} \\ \underline{\dot{\theta}}_i, & \underline{\dot{\theta}}_{i,\text{min}} \leq z \leq \underline{\dot{\theta}}_{i,\text{max}} \\ \underline{\dot{\theta}}_{i,\text{min}}, & z < \underline{\dot{\theta}}_{i,\text{min}} \end{cases}$$

where $\dot{\theta}_{i,\text{max}}$ represents the angular velocity limit of the $i$th joint.

In Fig. 2, $(U_1, U_2)$ represents the position command in Cartesian coordinates, and $(\dot{U}_1, \dot{U}_2)$ represents the corresponding position command in joint coordinates, which is input to the servo controller. $K_{pi}$ transforms position error into the velocity command and $K_{vi}$ transforms velocity error into the acceleration command, where $i = 1, 2$ represents the joint.

Servo controller parameters $K_{pi} = 17 \text{ l/s}, K_{vi} = 15 \text{ l/s}, K_{vi} = 102 \text{ l/s}$, and $K_{vi} = 90 \text{ l/s}$ represent the SCARA AR-M270 system. When the operation is within the bounds of torque saturation, the dynamics of each independent joint are given by

$$\frac{\ddot{\theta}_i(s)}{U_i(s)} = G(s) = \frac{K_{pi} K_{vi}}{s^2 + K_{vi} s + K_{pi} K_{vi}}.$$  \hspace{1cm} (7)

### III. ENHANCED CONTOUR CONTROL UNDER TORQUE CONSTRAINT

#### A. Methodology of Trajectory Generation

1) Sharpness Elimination at Corners: At the corners of an objective trajectory, velocity and acceleration demands become excessive. Consequently, torque saturation is frequently observed. To eliminate this occurrence, sharp corners are rounded up by substituting of circular arcs, as shown in Fig. 3(a). In Fig. 3(a), $\triangle RST \cong \frac{(\alpha_1 - \alpha_2)}{2}$. Referring to $\triangle RST$, the trigonometric relationship between the radius of arc $r$ and introduced error $\epsilon$ is given by $r/S/ST \cong r/(r + \epsilon) = \cos \left[ \frac{(\alpha_1 - \alpha_2)}{2} \right]$. Then, $r$ can be expressed in terms of $\epsilon$ as given by

$$r = \frac{\epsilon \cos \left[ \frac{(\alpha_1 - \alpha_2)}{2} \right]}{1 - \cos \left[ \frac{(\alpha_1 - \alpha_2)}{2} \right]}.$$  \hspace{1cm} (8)

2) Determination of Tracking Velocity at Corners: The tracking velocity profile is modified as shown in Fig. 3(b), where the circular corner is traversed with a low velocity $v_e$. The entire trajectory is represented by segments as marked by $t_i$, where $i = 0, 1, \ldots, 7$. According to the trajectory geometry and tracking velocity profile shown in Fig. 3(a) and (b), the generated trajectory is given by

$$\begin{align*}
    x(t) &= \begin{cases}
    A_{\text{max}} r^2 \cos \alpha_1/2, & 0 \leq t < t_1 \\
    x(t_1) + v(t - t_1) \cos \alpha_1, & t_1 \leq t < t_2 \\
    x(t_2) + [v(t - t_2) - A_{\text{max}} (t - t_2)^2/2] \cos \alpha_1, & t_2 \leq t < t_3 \\
    x(t_3) + r \left[ \sin \alpha_1 + v_r(t - t_3)/r - \sin \alpha_1 \right], & t_3 \leq t < t_4 \\
    x(t_4) + v_r(t - t_4) + A_{\text{max}} (t - t_4)^2/2 \cos \alpha_2, & t_4 \leq t < t_5 \\
    x(t_5) + v_r \cos \alpha_2, & t_5 \leq t < t_6 \\
    x(t_6) + v(t - t_6) - A_{\text{max}} \cos \alpha_1 (t - t_6)^2/2, & t_6 \leq t \leq t_7 
    \end{cases}
\end{align*}$$

$$y(t) = \begin{cases}
    A_{\text{max}} r^2 \sin \alpha_1/2, & 0 \leq t < t_1 \\
    y(t_1) + v(t - t_1) \sin \alpha_1, & t_1 \leq t < t_2 \\
    y(t_2) + [v(t - t_2) - A_{\text{max}} (t - t_2)^2/2] \sin \alpha_1, & t_2 \leq t < t_3 \\
    y(t_3) - r \left[ \cos \alpha_1 + \cos \alpha_1 + v_r(t - t_3)/r \right], & t_3 \leq t < t_4 \\
    y(t_4) + v_r(t - t_4) + A_{\text{max}} (t - t_4)^2/2 \sin \alpha_2, & t_4 \leq t < t_5 \\
    y(t_5) + v_r \sin \alpha_2, & t_5 \leq t < t_6 \\
    y(t_6) + v(t - t_6) - A_{\text{max}} \sin \alpha_2 (t - t_6)^2/2, & t_6 \leq t \leq t_7 
    \end{cases}$$  \hspace{1cm} (9)

where $A_{\text{max}}$ is the specification for maximum Cartesian acceleration.

3) Construction of Trajectory Corners for Different Conditions: Construction of corners is application oriented. There are two major categories of applications found in most industries. In the first category, the maximum tracking error $\epsilon_{\text{max}}$ is prespecified. In the second category, the minimum tracking velocity denoted by $v_{\text{r, min}}$ is specified.

Category 1 (given $\epsilon_{\text{max}}$): The tracking error at the corner should bound to its maximum value $\epsilon_{\text{max}}$ as given by $\epsilon \leq \epsilon_{\text{max}}$. Then, the radius of the circular arc is constrained according to $r \leq \epsilon_{\text{max}} \cos \left[ \frac{(\alpha_1 - \alpha_2)}{2} \right] / (1 - \cos \left[ \frac{(\alpha_1 - \alpha_2)}{2} \right])$. Centripetal acceleration along the circular arc is $v_e^2/r$. The maximum Cartesian acceleration $A_{\text{max}}$ is already known. Therefore, the maximum...
centripetal acceleration is given by $v_c^2/r = A_{\max}$. Tangential velocity $v_c$ is then determined by $v_c = \sqrt{A_{\max}r}$.

Category 2 (given $v_{c, \min}$): The constraint for tracking velocity is given by $v_c \geq v_{c, \min}$. Then, the radius of the circular arc is constrained according to $r \geq (v_{c, \min}^2/(2 \cos((\alpha_1 - \alpha_2)/2)))/A_{\max}$. Finally, $\epsilon$ is given by $\epsilon \geq (v_{c, \min}^2 (1 - \cos((\alpha_1 - \alpha_2)/2)))/A_{\max} \cos((\alpha_1 - \alpha_2)/2))$. The proposed trajectory in (9) and (10) can be numerically realized with the values determined for $A_{\max}$, $r$, and $v_c$, as described above.

B. Compensation for System Delay Dynamics

Joint coordinate trajectories are compensated for system delay dynamics, by the modified taught data (MTD) algorithm [7], which is illustrated in Fig. 4. The MTD algorithm is a feedforward compensator, which improves transient performance of joint dynamics. The compensator dynamics are given by $F_i(s) = -\gamma(s + K_p)/K_p(s - \gamma)$, where $\gamma$ is the pole of the compensator. The pole $\gamma$ should be selected appropriately, so that a fast stable system is realized. As a constraint inequality for stability, $\gamma \leq -K_p$. The exact value of $\gamma$ has to be determined from a computer simulation which evaluates transient performance of joint dynamics bound to rated motor speed. Since sampling time $\tau_s$ is fixed, the time constant of the modified system $1/\gamma$ should be bound to $1/\gamma > 10\tau_s$ to guarantee that waveform deterioration is eliminated. A detailed explanation of the MTD algorithm can be found in [7].

IV. RESULTS AND DISCUSSION

A. Implementation of the Proposed Contour Control Method

Referring to Figs. 1 and 4, the schematic of the contour control experimental setup is illustrated in Fig. 5.

The limits for joint acceleration for the two joints were $\dot{\theta}_{1, \max} = 1.432 \text{ rad/s}^2$ and $\dot{\theta}_{2, \max} = 1.732 \text{ rad/s}^2$, respectively. The right angular corner with vertices (0.2 m, 0.2 m), (0.25 m, 0, 2 m), and (0.25 m, 0.25 m) was selected as the objective trajectory. Maximum error $\epsilon_{\max}$ was set to 0.5 mm. According to category 1, $r = 1.2 \text{ mm}$ and $v_c = 0.05 \text{ m/s}$ with $A_{\max} = 0.05 \text{ m/s}^2$. The corresponding trajectory segmentation was implemented with time stamped at $t_0 = 0.000 \text{ s}$, $t_1 = 0.100 \text{ s}, t_2 = 0.988 \text{ s}, t_3 = 1.038 \text{ s}, t_4 = 1.110 \text{ s}, t_5 = 1.160 \text{ s}, t_6 = 2.048 \text{ s}$, and $t_7 = 2.148 \text{ s}$.

B. Contouring Results

Fig. 6(a) and (b) illustrates contouring results of the conventional and proposed methods. In the conventional method, the reference input generator carries out only sampling of the objective trajectory at a uniform tracking velocity along the entire path. Simulation and experimental results are comparable for both methods. Under conventional control, a significant overshoot occurs, as indicated by $\rightarrow A$ in Fig. 6(a). This overshoot can be approximated to 2 mm. In addition, significant contouring fluctuations are observed near the corner. In contrast, under the proposed control, overshooting is fully eliminated and the existing contour deteriorations are minute. Simulation results of joint accelerations are illustrated in Fig. 7(a)–(d). Under conventional control, joint 1 shows torque saturation, indicated by $\rightarrow B$ in Fig. 7(a), which lasts approximately 0.2 s, whereas joint 2 saturates for 0.1 s as indicated by $\rightarrow C$ in Fig. 7(b). In contrast, the proposed control causes no torque saturation, except a momentary demand of maximum torque at the trajectory corner, as indicated by $\rightarrow D$ in Fig. 7(c). Fig. 8(a) and (b) illustrates tracking velocity results of the conventional and proposed methods. Under conventional control, significant velocity overshoot is observed at the trajectory corner, which is indicated by $\rightarrow E$ in Fig. 8(a). In contrast, the proposed method does not cause such behavior.
C. Analysis of System Robustness

The behavior of contouring error $E_c$ was obtained by a simulation study of contour performance, in response to a change of servo parameter settings. The delay compensator was set to nominal servo parameter settings, whereas the servo controller settings of the two joints were changed in 5% steps from −70% to 130% of the nominal settings. The corresponding contouring performance was obtained as shown in Fig. 9(a). Each solid line illustrates the following trajectory obtained for a particular offset of servo parameters $K_{pv}$, in steps of 5%. The offset of servo parameters is denoted by $\Delta K_{pv}$ and is defined by $\Delta K_{pv} = (K_{pv} - K_{pv}^*)/K_{pv}^*$, where $K_{pv}$ and $K_{pv}^*$ refer to the position loop gains of the modification term and servo controller, respectively. When $K_{pv} < K_{pv}^*$, the following trajectory shifts inward, as can be seen in Fig. 9(a). Therefore, under this condition, contouring performance can be evaluated by the amount of inward shift from the particular trajectory obtained with the nominal settings. On the other hand, when $K_{pv} > K_{pv}^*$, the following trajectory shifts outward, resulting in an overshoot. Under this condition, contouring performance is evaluated by the amount of overshoot.

Fig. 9(b) illustrates the corresponding contouring error variation which was constructed from Fig. 9(a). It shows that contouring error increases with the decrease of servo parameters, whereas overshoot increases with the increase of servo parameters.

D. Discussion

Torque saturation indicated by $\rightarrow$ B in Fig. 7(a) causes velocity overshoot indicated by $\rightarrow$ E in Fig. 8(a). Such velocity overshoot consequently results in significant contour deteriorations in the following trajectory, as indicated by $\rightarrow$ A in Fig. 6(a).

Robustness of the proposed contour control system has been guaranteed. A maximum overshoot of 0.17 mm is introduced for a +30% change in position loop gain. Nevertheless, 0.17-mm overshoot for a 30% change in system parameters could be accepted for all or most industrial applications. Contouring error for a −30% change in system parameters does not exceed 0.6 mm. For most frequent small changes of system parameters, the corresponding contouring error or overshoot could be simply neglected.

In [8], contour control under torque constraint was realized for vertical-type articulated robot arms in which the position loop of the servo controller was open. In this paper, the method has been extended to SCARA robot where position loop of servo controller is closed.

In this paper, the proposed method has been realized for two-dimensional applications. Its extension to more realistic three-dimensional applications is possible on the basis of the following three guidelines.

1) The third joint of the SCARA is a prismatic joint with a vertical axis. For three-dimensional applications, this joint can be manipulated. Since the first two joints are rotary joints with vertical axes, operation of the prismatic joint has no coupling torque disturbances. Therefore, the third joint can be treated as another independent joint.

2) Almost all practical trajectories can be decomposed into a connected sequence of straight lines and corners. Therefore, trajectory generation can be carried out by parts, in Cartesian space. A three-dimensional corner, for example, can be again decomposed into its horizontal projection and vertical stretch. The prismatic joint of the SCARA robot can realize the vertical stretch independently from the horizontal projection, which could be tracked by the two rotary joints the same way as described above.

3) Even with articulated-type robot arms, on the assumption that joints have independent dynamics, three joints could be served in the same way as explained for the two links. The trajectory addressed in this paper is two-dimensional, however, a three-dimensional trajectory can be decomposed into a sequence of straight lines and two-dimensional corners that lay on different planes. Then, the solution could be applied on each segment in the sequence in order to contour a three-dimensional trajectory.

V. CONCLUSIONS

Contour control performance of industrial robot arms under torque constraint has been enhanced by the proposed method. Optimum avoidance of torque saturation and delay dynamics compensation have been successfully implemented and reduced into a single offline trajectory generation algorithm. Therefore, the proposed method can be conveniently implemented in industrial robot arms. Extension guidelines for the proposed method to address more realistic three-dimensional applications have also been discussed briefly.

Being offline, the necessary hardware changes for the implementation of the proposed method are minute and easily carried out. Moreover, the proposed trajectory generation method is highly flexible and could be easily customized for a particular application, in that it is always possible to figure out the optimum trajectory for a given contouring operation. Since torque saturation is avoided, the utility of industrial robot arms could be significantly improved by the proposed method.

REFERENCES


