TRAJECTORY PLANNING FOR ROBOT MANIPULATORS

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Based on MSc Research by Chinthaka Porawagama

Industrial Robotics Involves in
- Pick-and-place operations
- Assembling operations
- Loading and stacking
- Automated welding, etc.

Proper motion planning is needed in these applications

Path Definition

“Expressing the desired positions of a manipulator in the space, as a parametric function of time”


Trajectory Planning

Manipulators with multi degree of freedom for accomplishing various complex manipulation in the work space

Path: only geometric description
Trajectory: timing included
Trajectory Planning

Task to Trajectory

Joint Space Vs Operational Space

- **Joint-space description:**
  - The description of the motion to be made by the robot by its joint values.
  - The motion between the two points is unpredictable.

- **Operational space description:**
  - In many cases operational space = Cartesian space.
  - The motion between the two points is known at all times and controllable.
  - It is easy to visualize the trajectory, but it is difficult to ensure that singularity does not occur.

Planning in Operational Space

Sequential motions of a robot to follow a straight line.

- Cartesian-space trajectory
  - The trajectory specified in Cartesian coordinates may force the robot to run into itself, and
  - the trajectory may requires a sudden change in the joint angles.
Planning in Operational Space

- Calculate path from the initial point to the final point.
- Assign a total time $T_{path}$ to traverse the path.
- Discretize the points in time and space.
- Blend a continuous time function between these points
- Solve inverse kinematics at each step.

Advantages
- Collision free path can be obtained.

Disadvantages
- Computationally expensive due to inverse kinematics.
- It is unknown how to set the total time $T_{path}$.

Planning in Joint Space

- Calculate inverse kinematics solution from initial point to the final point.
- Assign total time $T_{path}$ using maximal velocities in joints.
- Discretize the individual joint trajectories in time.
- Blend a continuous function between these points.

Advantages
- Inverse kinematics is computed only once.
- Can easily take into account joint angle, velocity constraints.

Disadvantages
- Cannot deal with operational space obstacles.

Types of Motion

1. Point to point motion:
   - End effector moves from a start point to end point in work space
   - All joints' movements are coordinated for the point-to-point motion
   - End effector travels in an arbitrary path

2. Motion with Via Points
   - End effector moves through an intermediate point between start and end
   - End effector moves through a via point without stopping

Joint Space Planning

Point to point motion:
"Describing of joints' motions from start to end by smooth functions"

1. Inverse kinematics of start and end points (A & B)
2. Joint angles for start and end points

Task Space

Parametric Representation

Arbitrary path in the task space
Joint Space Planning

3. Interpolation of start and end joint angles by smooth functions
   \( \theta_1(t) : \theta_{1A} \rightarrow \theta_{1B} \)
   \( \theta_2(t) : \theta_{2A} \rightarrow \theta_{2B} \)
   \( \theta_3(t) : \theta_{3A} \rightarrow \theta_{3B} \)

4. Joint space trajectories for each joint

Start point 'A'
\( \theta_{1A} \theta_{2A} \theta_{3A} \)

End point 'B'
\( \theta_{1B} \theta_{2B} \theta_{3B} \)

Arbitrary path in the task space

Task Space

Parametric Representation

Travel time \( T \)

\( \theta_1(t) \)
\( \theta_2(t) \)
\( \theta_3(t) \)

Smooth Motion ➔ Quality of Work

- Non smooth trajectories lead to
  - Vibration/jerk
  - Actuator saturation/Path deviation
  - Shorten manipulator lifetime
  - Poor quality in work

Uniform Velocity Trajectory

\[
q(t) = q^* + \frac{q^* - q^0}{t_g} t
\]

\[
\dot{q}(t) = \frac{q^* - q^0}{t_g}
\]

\[
\ddot{q}(t) = \begin{cases} 
\infty & t = 0, t_g \\
0 & 0 < t < t_g 
\end{cases}
\]

- Infinite accelerations at endpoints
- Discontinuous velocity when two trajectory segments are connected (at via points)

Triangular Velocity Trajectory

\[
q^* - q^0 = \frac{1}{2} q^*_g \left( \frac{t \min}{t_g} \right)
\]

\[
q(t) = \begin{cases} 
q^* + 0.5 q^*_g t^2 & 0 \leq t < 0.5 t_g \\
0.5(q^* + q^0) - 0.5 q^*_g t^2 & 0.5 t_g \leq t \leq t_g 
\end{cases}
\]

\[
\ddot{q}_{\min} = \begin{cases} 
\frac{4(q^* - q^0)}{t_g^2} & t \leq 0.5 t_g \\
- \frac{4(q^* - q^0)}{t_g^2} & 0.5 t_g \leq t \leq t_g 
\end{cases}
\]

- Acceleration discontinuities exist at endpoints and at the midpoint of the trajectory
Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends

- Zero acceleration in middle segments.
- Constant acceleration at end segments.

Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends

\[ \text{Zero acceleration in middle segments.} \]
\[ \text{Constant acceleration at end segments.} \]

Trapezoidal Velocity Trajectory: Linear Trajectory with Parabolic Blends

\[ \text{Constant acceleration at end segments.} \]

Trapezoidal Velocity Spline Trajectory

- Multi-stage linear parabolic blend (spline)

\[ \text{Multi-stage linear parabolic blend (spline)} \]
Cubic Polynomial Trajectory:
Improving smoothness

Joint Position
\[ q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \]  

Joint Velocity
\[ \dot{q}(t) = a_1 + 2 a_2 t + 3 a_3 t^2 \]

Joint Acceleration
\[ \ddot{q}(t) = 2 a_2 + 6 a_3 t \]

Joint Jerk
\[ \dddot{q}(t) = 6 a_3 \]

Boundary Conditions for position
\[ q_s = a_0 \]  
\[ q_s = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \]

Boundary conditions for speed
\[ \dot{q}_s = a_1 \]  
\[ \dot{q}_s = a_1 + 2 a_2 T + 3 a_3 T^2 \]

What about joint accelerations?
\[ \ddot{q}_s = 2 a_2 \]  
\[ \ddot{q}_s = 2 a_2 + 6 a_3 T \]

Cubic Polynomial with zero speed at end-points

Parameter Calculation
\[ (4), (6) \Rightarrow (5), \]  
\[ q_s = q_e + \dot{q}_e T + a_2 T^2 + a_3 T^3 \]
\[ \dot{q}_s = \dot{q}_e + 2 a_2 T + 3 a_3 T^2 \]
\[ q_s - q_e - \dot{q}_e T = a_2 T^2 + a_3 T^3 \]  
\[ \dot{q}_s - \dot{q}_e = 2 a_2 T + 3 a_3 T^2 \]
\[ a_2 = \frac{3(q_s - q_e - \dot{q}_e T) - (\dot{q}_s - \dot{q}_e)}{3T^2 - 2T} \]
\[ a_3 = \frac{(\dot{q}_s - \dot{q}_e) - 2(q_s - q_e - \dot{q}_e T)}{3T^2 - 2T^3} \]
Cubic Polynomial with zero speed at end-points

Cubic Splines: Piecewise Cubic Polynomial Fitting

PTP with one Via Point

- Plan the trajectory

<table>
<thead>
<tr>
<th></th>
<th>start</th>
<th>via point</th>
<th>end</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pos</td>
<td>given</td>
<td>To be continuous</td>
<td>given</td>
</tr>
<tr>
<td>Vel</td>
<td>given=0</td>
<td>To be continuous</td>
<td>given=0</td>
</tr>
<tr>
<td>Acc</td>
<td>given=0</td>
<td>Not constrained</td>
<td>given=0</td>
</tr>
</tbody>
</table>

8 boundary conditions (equations)

- Stitching cubic polynomials together

Cubic Spline Trajectory

\[
\begin{align*}
q(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\
v(t) &= a_1 + 2a_2 t + 3a_3 t^2 \\
n(t) &= 2a_2 + 6a_3 t
\end{align*}
\]
5th Order Polynomial: Controls higher order derivatives of the trajectory

\[ q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \]

\[ \dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \]

\[ \ddot{q}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 \]

\[ \dddot{q}(t) = 6a_3 + 24a_4 t + 60a_5 t^2 \]